Information Sharing Policy and Inter-dealer OTC Markets

Yilin Wang*

November 26, 2019

Abstract

Many decentralized over-the-counter (OTC) markets have recently become subject to new regulations requiring transparency. I build up an information model that features bilateral trade in double auction, endogenous public signal, and inter-dealer network formation to study the effect of TRACE on the inter-dealer markets. In the trading stage, I study the private information diffusion process and endogenize the public information contained in the disseminated trading price. I show that in markets with a relatively low degree of information asymmetry, post-trade transparency makes the adverse selection more severe and reduces the surplus from asset reallocation between dealers, and thus hurts the inter-dealer network formation. Investors are more likely to be symmetrically uninformed about thinly traded bonds. The empirical results provide evidence for that and show TRACE has a significant negative effect on the inter-dealer trading frequency for thinly traded bonds.

JEL Classifications: G14, D82, D85

*Economics Department, University of California, Los Angeles. Email: wangyilin@ucla.edu. I am grateful to Pierre-Olivier Weill, Andrew Atkeson, Tomasz Sadzik, Liyan Yang, Kei Kawakami, and numerous seminar participants at UCLA and the FTG Summer School at Wharton for helpful comments and discussions. I owe Thomas R. Covert special thanks for sharing with me the electronic version of the bond list of each phase and the FINRA50 bond list. All errors are my own.

1 Introduction

Many regulations aim at enhancing information sharing in financial markets. In 2002, FINRA (Financial Industry Regulatory Authority) began requiring the timely public dissemination of post-trade price and volume information for the U.S. corporate bond market through TRACE (FINRA's Trade Reporting and Compliance Engine). TRACE has become the template for increased transparency in other over-the-counter financial markets. FINRA expanded TRACE to several other asset classes, including Agency-Backed Securities, since 2010, and Asset-Backed Securities, since 2011. Title VII of Dodd-Frank Wall Street Reform has required that swaps (including CDS, interest rate swaps, collateralized debt obligations, and other derivatives) adopt TRACE-like post-trade transparency since 2011. European MiFID II/R regulations mimic TRACE for European corporate bonds and were implemented in 2018.

Asquith, Covert, and Pathak (2019) also examine the effect of TRACE on thinly traded, high-yield bonds. They find after transparency, while trading costs declined significantly for the entire bond market, there was a significant decline in the number of trades for thinly traded bonds. Many of the securities markets that are newly subject to transparency are thinly traded. Their empirical results support the view that not every segment of a security market should be subject to the same degree of mandated transparency.

This suggests that theoretical models are needed to study the non-uniform effects of transparency policy on the different segments of OTC markets. In this paper, I build up a model that features bilateral trade in double auction between dealers, endogenous public signal, and inter-dealer network formation. My model shows that in markets with a relatively low degree of information asymmetry, post-trade transparency makes the adverse selection more severe and reduces the surplus from asset reallocation between dealers. In those markets, TRACE hurts the network formation and lowers inter-dealer trading frequency.

In the trading stage, I study a line trading network of risk-neutral dealers. For the sake of tractability, I assume the first dealer in the line network has private information about the asset payoff. The dealers have their own private values that are their private information. The trade sequentially takes place between the first dealer and the second dealer, then between the second dealer and the third dealer, etc. Each dealer chooses its demand functions for the transactions, to maximize its expected profits, given its private value and its information about the asset payoff, and they take their price impacts into consideration. I characterize the linear Bayesian Nash equilibrium and show that in equilibrium, the information asymmetry between two consecutive dealers before their trade decides how aggressively they trade on their private values, and thus decides the private information diffusion process and the surplus of asset reallocation between them.

To study the effect of TRACE on the inter-dealer markets, I introduce the public signal to the model. For simplicity, I consider the release of just one public signal. I study two different information structures of the public signal. I first show the effect of transparency on dealers' trading surplus when the public signal is exogenous. Then I show the effect of transparency on dealers' trading surplus after endogenizing the public signal as the disseminated price of TRACE. Contrast to traditional wisdom, my model shows that the disseminated trading price as the public signal is not desirable for some markets. The key difference between the endogenous and the exogenous public signal is that the endogenous public signal is both about asset payoff and about previous dealers' private values. The information about private values can allow the more informed dealer to filter the signal more effectively.

In the network formation stage, I study a network formation model to endogenize the inter-dealer network structure. The network formation stage is before the trading stage; thus, dealers trading surplus in the trading stage affects their network formation decision. In the network formation model, the dealer with private information about the asset payoff bargains with the second dealer over how to split the link formation cost; the second dealer who forms the link bargains with the third dealer over how to split the link formation cost, etc. In equilibrium, the surplus of each formed link would spread over all the dealers before this link and affect their network formation decision. Thus the effect of TRACE on downstream dealers' trading profit would affect the upstream dealers' link formation decision. The lower trading profits of downstream dealers could hurt the the formation of the whole network. This model implies that in markets with low information asymmetry, TRACE hurts the network formation and thus lowers inter-dealer trading frequency. The network formation game also implies that TRACE could reduce the trading frequency of the upstream dealers as well, because they benefit less from the downstream dealers' trading profits if they form the link to initiate the trade.

On the empirical side, I study the effect of TRACE on corporate bond inter-dealer markets, by utilizing the Academic Corporate Bond TRACE data and the Mergent FISD database. The argument is that if a bond is very actively traded, then although investors are more likely to know more about them, they are also more likely to be asymmetrically informed; vice versa, if a bond is thinly traded, then investors are more likely to be symmetrically uninformed. So the model predicts that TRACE would have negative effects on inter-dealer trading activity for thinly traded bonds. The empirical findings in this paper show that TRACE has significant negative effect on the inter-dealer trading frequency only for thinly traded bonds. To be more specific, DID estimates show that TRACE has a non-negative or positive effect on the inter-dealer trading frequency of actively traded bonds, Phase 2 bonds, and Phase 3B bonds. In contrast, TRACE significantly lowers the inter-dealer trading frequency of Phase 3B bonds, which are the most thinly traded corporate bonds. It is a warning for the widespread implementation of TRACE, as many of the assets that are newly subject to transparency are similar to Phase 3B bonds as they are also thinly traded.

Related Literature

This paper follows the double auction literature when modeling the trading game, e.g. Kyle (1989) and Malamud and Rostek (2017). My work adds to the growing literature on network studies in financial markets. The application of network theory to financial markets has only just begun. Blume et al. (2009) and Gale and Kariv (2007) study how a network intermediates trades in a decentralized market. Gofman (2011) assesses the efficiency of resource allocation through the trading network in an OTC market. Malamud and Rostek (2017) develop a general framework for studying dealers' strategic interactions in decentralized markets. Many past studies also focus on information acquisition from a network and its impact on financial markets. Han and Yang (2013) extend the rational expectation equilibrium model to study the information network in a financial market. Babus and Kondor (2018) and Babus, Kondor and Wang (2019) study information transmission through inter-dealer networks in the OTC markets by extending the model in Vives (2011) to games in networks. In addition to using network models to study OTC markets, others apply network models to the interbank market to analyze contagion risk in the banking system, e.g. Babus (2016) and Elliott, Golub, and Jackson (2013).

Most models of OTC markets are based on search and bargaining. Duffie, Garleanu, and Pedersen (2005, 2007) study how search and bargaining determine prices in the OTC markets. Atkeson, Eisfeldt, and Pierre-Olivier Weill (2015) study how market entry costs help determine the structure of OTC trading, and thereby prices charged in OTC trading.

There is a branch of theoretical work on the impact of transparency on trading behavior in financial markets. Pagano and Roell (1996) argue that well-informed dealers may be able to extract rents from less well-informed customers in an opaque market. Bloomfield and O'Hara (1999) show that transparency can reduce market-makers incentives to supply liquidity, if market makers have more difficulty unwinding inventory following large traders. Naik, Neuberger, and Viswanathan (1999) show that transparency can improve dealers' ability to share risks, which decreases their inventory costs and therefore customers' costs of trading. Madhavan (1995) demonstrates hat dealers may prefer not to disclose trades because they benefit from the reduction in information. Some of this work highlights the downside of more transparency, but none of it studies the impact of transparency on network formation in the OTC market. My paper fills this gap in the literature.

The effect of TRACE on the US corporate bond market has been studied in some empirical work. Bessembinder, Maxwell, and Venkataraman (2006), focusing on Phase 1 only, which covered investment grade and large issue bonds, document a reduction in trade execution costs, estimated using a structural model. Edwards, Harris, and Piwowar (2007) and Hotchkiss, Goldstein, and Sirri (2007), both using investment grade bonds in Phase 2 TRACE data, report no effect on the trading activity and a decline in transaction costs. Asquith, Covert, and Pathak (2019) find that even though trading costs decrease significantly across all types of bonds, transparency effects are not uniform across different segments of the bond market; after transparency, there is a significant decline in the number of trades for Phase 3B bonds, which are far more likely to be lower rated high yield bonds, while transparency has a limited impact on the trading activity of investment grade bonds and the most frequently-traded bonds. Bessembinder and Maxwell (2008) survey dealers and report that bond dealers almost universally perceive that trading became more difficult after TRACE.

There is a set of studies on municipal bonds. On January 31, 2005, the Municipal Securities Rulemaking Board (MSRB) started requiring that information about trades in municipal bonds be reported within 15 minutes, similar to TRACE. Prior to that dissemination, Green, Hollifield, and Schurhoff (2007a) find significant price dispersion in new issues of municipal bonds, which they attribute to the decentralized and opaque market design. Schultz (2012) compares price dispersion at the offering date for municipal bonds before and after this change and finds that it falls sharply. Brancaccio, Li, and Schurhoff (2017) show that the MSRB transparency rule reduced trading volume in uninsured bonds, but not in insured bonds.

Some papers study another kind of information sharing, that is, dealers' sharing of clients' bid information. Di Maggio, Franzoni, Kermani, and Sommavilla (2017) document that equity dealers share clients' bid information with other clients. Boyarchenko, Lucca and Veldkamp (2018) use a quantitative model to study the effect of dealers' information sharing of clients' order order flow information in a centralized market.

Some empirical papers study the effect of public information on information asymmetry. Kim and Verrecchia (1994, 1997) suggest that public information releases may actually increase information asymmetry if market participants differ in their ability to interpret the news. Findings in Green (2004) indicate that the release of public information raises the level of information asymmetry in the government bond market. The results indicate that information asymmetry in the government bond market arises not from the absence of relevant public information, but rather the ability of market participants to interpret the information.

My work also contributes to the literature that studies endogenous network structure. There are other models that study the network formation in financial markets for different markets or from different perspectives, see Babus (2016), Zhong and Kawakami (2016), and Chang and Zhang (2019).

The paper is organized as follows. The following section introduces the model setup of trading in the inter-dealer markets and the equilibrium concept of the trading game. In Section 3, I derive the Linear Bayesian Nash equilibrium of the trading game in opaque markets and show the information asymmetry decides dealers' surplus from asset reallocation. In Section 4, I introduce the public signal to the model; I study two different information structures of the public signal and show that the TRACE transparency would increase the degree of information asymmetry and lower dealers' trading surplus in markets with a relatively low degree of information asymmetry. In Section 5, I propose a network formation model to study the effect of TRACE on the inter-dealer network structure. In Section 6, I do empirical analysis to study the effect of TRACE post-trade transparency on inter-dealer trading activity in different markets. In Section 7, I study variants of the baseline model and show that the results of the baseline model are robust. Finally, I conclude. Proofs and data cleaning procedures are in the appendix.

2 A Model of Trading in the Inter-dealer Markets

In this section I first describe the agents and the trading game they play, then I define the equilibrium of the trading game.

2.1 The Model Setup

2.1.1 Information structure

We consider an economy with n risk-neutral dealers that trade bilaterally a divisible risky asset. The asset is in zero net supply.

Dealer i's value of the asset is

$$\theta_i = \theta + \eta_i,$$

where $\theta \sim \mathcal{N}(0, \sigma_\theta^2)$, $\eta_i \overset{i.i.d}{\sim} \mathcal{N}(0, \sigma_\eta^2)$, $\theta \perp \eta_i$. This implies that θ is normally distributed with a mean normalized to zero and a variance σ_θ^2, and η_i are drawn independently across dealers and from θ.

Each dealer i knows its own η_i, but does not know θ. Dealer 1 receives the private signal about θ,

$$s_1 = \theta + \varepsilon,$$

where $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$, $\theta \perp \varepsilon$, $\eta_i \perp \varepsilon$.

In this model without public signal, other dealers do not receive any exogenous information. I define opaque markets to be the markets without public signal. I will introduce the public signal to the model in Section 3.2.

2.1.2 Dealer's action and payoff

Let $p_{i-1,i}$ or $p_{i,i-1}$ denote the price at which trade takes place over link $(i-1, i)$.

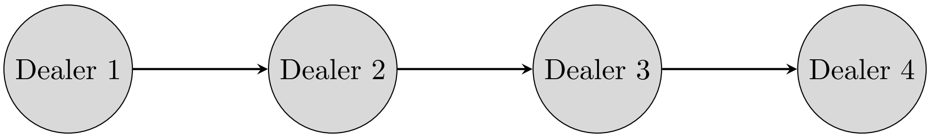

Figure 1: The inter-dealer network with $n = 4$

Suppose the network has one dealer, then there is no trade between dealers. Suppose the network has n dealers, then the trade takes place sequentially between dealer 1 and dealer 2, then between dealer 2 and dealer 3,...,between dealer $n-1$ and dealer n.

I assume there is demand from clients. The aggregate demand of clients on each link $(i, i+1)$ is $\beta p_{i,i+1}$, where $\beta < 0$. Clients' linear demand is micro-founded in the appendix.[1]

The demand function of dealer 1 is

$$Q^1_{1,2}(\eta_1, s_1, p_{1,2}),$$

where $Q^1_{1,2}$ maps η_1, s_1, and $p_{1,2}$ into the quantity it wishes to trade on the link (1,2).

The demand functions of dealer $i \in \{2, .., n-1\}$ are

$$\left(Q^i_{i-1,i}(\eta_i, p_{i-1,i}), Q^i_{i,i+1}(\eta_i, p_{i-1,i}, p_{i,i+1})\right),$$

where $Q^i_{i-1,i}$ maps η_i, $p_{i-1,i}$ into the quantity it wishes to trade on the link $(i-1, i)$; and $Q^i_{i,i+1}$ maps η_i, $p_{i-1,i}$ and $p_{i,i+1}$ into the quantity it wishes to trade on the link $(i, i+1)$.

The demand function of dealer n is

$$Q^n_{n-1,n}(\eta_n, p_{n-1,n}),$$

where $Q^n_{n-1,n}$ maps η_n, $p_{n-1,n}$ into the quantity it wishes to trade on the link $(n-1, n)$.

OTC trading protocols do not typically involve the submission of full demand schedules. But as mentioned in Babus and Kondor (2019), generalized demand functions capture the repeated exchange of limit and market orders (i.e., the offer and acceptance

[1] I assume the clients are non-atomic and risk-neutral. They incur quadratic flow cost when trading. The quadratic flow cost is used in the models of Rostek and Weretka (2012) and Du and Zhu (2017). The fixed β for all the links is based on the assumption that clients only trade for liquidity needs. In Section 6.2, I relax this assumption and clients also trade for the asset payoff, and I show that the qualitative results in the baseline model still hold and the effect of transparency on dealers' trading surplus is amplified due to the effect of information asymmetry on clients' trading intensity.

of quotes) within a short time interval across fixed counterparties as a reduced-form price determination mechanism. [2]

The expected payoff of dealer 1 is

$$\mathbb{E}(\pi^1_{1,2}) = \mathbb{E}\big[Q^1_{1,2}(\eta_1, s_1, p_{1,2})(\theta_1 - p_{1,2})\big].$$

The expected payoff of dealer $i \in \{2, .., n-1\}$is

$$\mathbb{E}(\pi_i) = \mathbb{E}(\pi^i_{i-1,i}) + \mathbb{E}(\pi^i_{i,i+1}),$$

where

$$\mathbb{E}(\pi^i_{i-1,i}) = \mathbb{E}[Q^i_{i-1,i}(\eta_i, p_{i-1,i})(\theta_i - p_{i-1,i})],$$

$$\mathbb{E}(\pi^i_{i,i+1}) = \mathbb{E}[Q^i_{i,i+1}(\eta_i, p_{i-1,i}, p_{i,i+1})(\theta_i - p_{i,i+1})].$$

The expected payoff of dealer n is

$$\mathbb{E}(\pi^n_{n-1,n}) = \mathbb{E}\big[Q^n_{n-1,n}(\eta_n, p_{n-1,n})(\theta_n - p_{n-1,n})\big].$$

2.2 Equilibrium Concept

I consider linear equilibria of the game, defined as follows.

Definition 1. *A Linear Bayesian Nash equilibrium of the trading game is a vector of dealers' linear demand functions*

$$\Big\{Q^1_{1,2}(.), \big(Q^2_{1,2}(.), Q^2_{2,3}(.)\big), ..., \big(Q^{n-1}_{n-2,n-1}(.), Q^{n-1}_{n-1,n}(.)\big), Q^n_{n-1,n}(.)\Big\},$$

such that

(1) $Q^1_{1,2}(\eta_1, s_1, p_{1,2})$ *solves the problem*

$$\max_{Q^1_{1,2}} \mathbb{E}\big[Q^1_{1,2}(\theta_1 - p_{1,2})|\eta_1, s_1\big],$$

(2) $\big(Q^i_{i-1,i}(\eta_i, p_{i-1,i}), Q^i_{i,i+1}(\eta_i, p_{i-1,i}, p_{i,i+1})\big)$ *for* $i \in \{2, .., n-1\}$ *solves the problem*

$$\max_{(Q^i_{i-1,i}, Q^i_{i,i+1})} \mathbb{E}[Q^i_{i-1,i}(\theta_i - p_{i-1,i})|\eta_i] + \mathbb{E}[Q^i_{i,i+1}(\theta_i - p_{i,i+1})|\eta_i, p_{i-1,i}],$$

[2] In Section 6.2, I explicitly model the price-discovery process. The price discovery game shows that the equilibrium prices and quantities of my baseline model can be found via an iterative, decentralized process.

(3) $Q^n_{n-1,n}(\eta_n, p_{n-1,n})$ *solves the problem*

$$\max_{Q^n_{n-1,n}} \mathbb{E}\big[Q^n_{n-1,n}(\theta_n - p_{n-1,n})|\eta_n\big],$$

(4) $p_{1,2}$, $p_{i,i+1}$ *for* $i \in \{2, ..., n-1\}$ *clear the market,*

$$Q^1_{1,2}(\eta_1, s_1, p_{1,2}) + Q^2_{1,2}(\eta_2, p_{1,2}) + \beta p_{1,2} = 0,$$

$$Q^i_{i,i+1}(\eta_i, p_{i-1,i}, p_{i,i+1})) + Q^{i+1}_{i,i+1}(\eta_{i+1}, p_{i,i+1}) + \beta p_{i,i+1} = 0.$$

Each dealer chooses its demand functions for the transactions, to maximize its expected profits, given its private value (for dealer 1, also given its private information) and given the demand functions chosen by its counter-parties. For each transaction, given both dealers' demand functions, the equilibrium price clears the market. Implicit in the definition of the equilibrium is that each dealer understands that she has a price impact when trading with the counterparties.

3 Equilibrium of the Trading Game

In this section, I derive the equilibrium of the trading game in opaque markets. I show the information asymmetry between dealers decide how aggressively they trade on their private values in equilibrium, and thus decide the surplus from asset reallocation between them.

The following proposition characterizes the trading behavior of dealer 1 and dealer 2, and the price function on the link (1, 2).

Proposition 1. *In the Linear Bayesian Nash equilibrium of the trading game between dealer 1 and 2, dealer 1's demand function is*

$$Q^1_{1,2} = a^1_{1,2}s_1 + b^1_{1,2}p_{1,2} + c^1_{1,2}\eta_1.$$

Dealer 2's demand function on link (1,2) is

$$Q^2_{1,2} = b^2_{1,2}p_{1,2} + c^2_{1,2}\eta_2.$$

Define

$$\kappa_1 \equiv \frac{\sigma^2_\eta}{Var(\theta) - Var(\theta|s_1)} = (1 + \frac{\sigma^2_\varepsilon}{\sigma^2_\theta})\frac{\sigma^2_\eta}{\sigma^2_\theta} := (1+\gamma)\varphi,$$

which is the inverse of information asymmetry between dealer 1 and 2 scaled by σ_η^2. *The degree of the interdependence between dealers' values is captured by* $\varphi \equiv \frac{\sigma_\eta^2}{\sigma_\theta^2}$. *The inverse of dealer 1's information precision is captured by* $\gamma \equiv \frac{\sigma_\varepsilon^2}{\sigma_\theta^2}$.

$a_{1,2}^1, b_{1,2}^1, b_{1,2}^2, c_{1,2}^1, c_{1,2}^2$ *have closed-form solution*

$$b_{1,2}^1 = b_{1,2}^2 + \beta = \beta\kappa_1,$$
$$c_{1,2}^1 = c_{1,2}^2 = -\beta\kappa_1,$$
$$\frac{c_{1,2}^1}{a_{1,2}^1} = \frac{\kappa_1}{\varphi}.$$

The price function is

$$p_{1,2} = \frac{\varphi}{2\kappa_1}s_1 + \frac{\eta_1 + \eta_2}{2}.$$

In order to solve for the equilibrium of the trading game between dealer 2 and 3, we first analyze the information dealer 2 gets from trading with dealer 1. For dealer 2, the market clearing condition for link (1,2) implies

$$Q_{1,2}^2 + a_{1,2}^1 s_1 + b_{1,2}^1 p_{1,2} + c_{1,2}^1 \eta_1 + \beta p_{1,2} = 0,$$

thus

$$p_{1,2} = -\frac{a_{1,2}^1 s_1 + c_{1,2}^1 \eta_1}{b_{1,2}^1 + \beta} - \frac{Q_{1,2}^2}{b_{1,2}^1 + \beta} := I_2 + \lambda_{1,2}^2 Q_{1,2}^2$$

Dealer 2 knows his or her own demand, so dealer 2 can infer the intercept I_2. Thus, dealer 2 can learn the private signal s_2,

$$s_2 \equiv s_1 + \frac{c_{1,2}^1}{a_{1,2}^1}\eta_1 = s_1 + \frac{\kappa_1}{\varphi}\eta_1.$$

Similarly, I define s_i ($i \in \{2, ..., n-1\}$) to be the signal dealer i learns from price $p_{i-1,i}$. Following the same equilibrium characterization procedure of Proposition 1, the trading game between dealer 2 and 3, ..., dealer $n-1$ and n can be solved, as shown in the following lemma.

Lemma 1. *In the Linear Bayesian Nash equilibrium of the trading game between dealer* i *and* $i+1$ ($i \in \{2, 3, ..., n-1\}$), *dealer* i*'s demand function can be expressed as*

$$Q_{i,i+1}^i = a_{i,i+1}^i s_i + b_{i,i+1}^i p_{i,i+1} + c_{i,i+1}^i \eta_{i,i+1},$$

where

$$s_i = \frac{2\kappa_{i-1}p_{i-1,i} - \kappa_{i-1}\eta_i}{\varphi}.$$

Dealer $i+1$'s demand function on link $(i,i+1)$ is

$$Q_{i,i+1}^{i+1} = b_{i,i+1}^{i+1} p_{i,i+1} + c_{i,i+1}^{i+1} \eta_{i+1}.$$

Define

$$\kappa_i \equiv \frac{\sigma_\eta^2}{Var(\theta) - Var(\theta|s_i)},$$

which is the inverse of information asymmetry between dealer i and $i+1$ scaled by σ_η^2. $a_{i,i+1}^i, b_{i,i+1}^i, b_{i,i+1}^{i+1}, c_{i,i+1}^i, c_{i,i+1}^{i+1}$ have closed-form solution

$$b_{i,i+1}^i = b_{i,i+1}^{i+1} + \beta = \beta\kappa_i,$$
$$c_{i,i+1}^i = c_{i,i+1}^{i+1} = -\beta\kappa_i,$$
$$\frac{c_{i,i+1}^i}{a_{i,i+1}^i} = \frac{\kappa_i}{\varphi}.$$

The price function is

$$p_{i,i+1} = \frac{\varphi}{2\kappa_i} s_i + \frac{\eta_i + \eta_{i+1}}{2}.$$

In Proposition 1 and Lemma 1, from $c_{i,i+1}^i = c_{i,i+1}^{i+1} = -\beta\kappa_i$, we can see in equilibrium how aggressively dealers trade on their private values is decided by the inverse of information asymmetry between two consecutive dealers on each link scaled by the variance of private values, $\frac{\sigma_\eta^2}{Var(\theta)-Var(\theta|s_i)}$; it is also decided by clients' trading intensity. This will create important implications for dealers' trading surplus, which will be shown later.

The following lemma generalizes the private information diffusion process over links.

Lemma 2. *In equilibrium, dealer $i+1$ learns s_i from trading with dealer i,*

$$s_{i+1} = s_1 + \frac{\kappa_1}{\varphi}\eta_1 + \frac{\kappa_2}{\varphi}\eta_2 + ... + \frac{\kappa_i}{\varphi}\eta_i.$$

Define

$$\kappa_{i+1} \equiv \frac{\sigma_\eta^2}{Var(\theta) - Var(\theta|s_{i+1})} = \kappa_i + \kappa_i^2.$$

From Lemma 2, we can see the information diffusion process is governed by the initial information asymmetry between dealer 1 and 2. If the degree of informational asymmetry between dealer 1 and 2 is lower, then the degree of information asymmetry between all the subsequent consecutive dealers will become lower before their trade. This is because a lower degree of information asymmetry between dealer 1 and 2 will make a dealer trade relatively more aggressively on private value than private signal, reflected by the ratio $\frac{c_1}{a_1} = \frac{\kappa_1}{\varphi}$. Thus dealer 2 learns less from the price between dealer 1 and 2 and the degree of information asymmetry between dealer 2 and 3 is lower as well, which makes dealer 2 trade relatively more aggressively on private value than the signal learned from trading with dealer 1, and so on.

Now I characterize dealers' trading profit using the result from Proposition 1 and Lemma 1. Dealers' profit from each link can be decomposed into two components: the surplus from asset reallocation between dealers and the profits from serving clients. Considering dealer $i+1$ demands $Q^{i+1}_{i,i+1}$ units of the asset, clients demand $\beta p_{i,i+1}$ units of the asset, dealer i supplies $Q^{i+1}_{i,i+1} + \beta p_{i,i+1}$ units of the asset. The trading profit of dealer i and $i+1$ is

$$\mathbb{E}[(Q^{i+1}_{i,i+1}(\eta_{i+1} - \eta_i)] + \mathbb{E}[\beta p_{i,i+1}(p_{i,i+1} - \theta - \eta_i)],$$

where the first component is the surplus from the asset reallocation between dealers, and the second component is the rent dealer i extracts from serving the clients.

The following lemma shows dealers' profit on each link.

Lemma 3. *In equilibrium, the ex-ante trading profit of dealer i and $i+1$ from link $(i, i+1)$ is*

$$\mathbb{E}(\pi^{i}_{i,i+1}) + \mathbb{E}(\pi^{i+1}_{i,i+1}) = -\beta\sigma^2_\eta\kappa_i + \frac{-\beta}{4}\frac{\sigma^2_\eta}{\kappa_i},$$

where $-\beta\sigma^2_\eta\kappa_i$ is the surplus from asset reallocation between dealer i and $i+1$, $\frac{-\beta}{4}\frac{\sigma^2_\eta}{\kappa_i}$ is dealer i's profit from serving clients.

Lemma 3 shows less severe adverse selection increases the surplus from asset reallocation between dealers. This is because the lower degree of information asymmetry will make dealers trade more aggressively on their private values, and thus increase the trading surplus from asset reallocation between them.

To link my theory to more observable characteristics of the inter-dealer markets documented in the empirical literature, I define the inter-dealer trading cost of dealer $i \in \{2, ..., n-1\}$ to be $|p_{i,i+1} - p_{i-1,i}|$. This definition of trading cost is similar to that in Li and Schurhoff (2019), which measures trading costs for investors by inter-dealer markups.

Lemma 4. *For $i \in \{2, ..., n-1\}$,*

$$Var(p_{i,i+1} - p_{i-1,i}) = \frac{\sigma_\eta^2}{4} \frac{\kappa_{i-1}}{\kappa_{i-1} + 1} + \frac{1}{4}\sigma_\eta^2,$$

thus

(1) Inter-dealer trading cost $\mathbb{E}(|p_{i,i+1} - p_{i-1,i}|) = \sqrt{\frac{2}{\pi}}\sqrt{Var(p_{i,i+1} - p_{i-1,i})}$ *is increasing in i.*

(2) The average inter-dealer trading cost $\frac{\sum_{i \in \{2,...,n-1\}} \mathbb{E}(|p_{i,i+1} - p_{i-1,i}|)}{n-2}$ *is increasing in n.*

Lemma 4 shows that the inter-dealer trading cost is increasing over links; thus, the average inter-dealer trading cost of the whole chain is increasing in number of dealers in the chain. This result is consistent with the empirical finding in Li and Schurhoff (2019) that average markups increase monotonically with the number of dealers intermediating the chain, and the evidence in Schultz (2012) on newly issued bonds, from 1.9% on average when one dealer is involved to 3.7% with seven dealers involved.

4 Exogenous and Endogenous Public Signal

In this Section, I first introduce the public signal to the trading game, and derive the equilibrium of the trading game. Using the equilibrium solution, I study in both cases how the information diffuses and how it affects dealers' trading surplus from asset reallocation between them. Then I study two different information structures of the public signal and highlight the different implications in these two scenarios.

4.1 Introduction of a Public Signal

For simplicity, I consider the release of just one public signal S. To allow delays in the information release, as is done in practice, I consider that the signal is made available to all traders after the trade between dealer $n_p - 1$ and n_p is done.

$$S = \theta + \varepsilon_S,$$

where $\varepsilon_S \sim \mathcal{N}(0, \sigma_p^2)$, $\theta \perp \varepsilon_S$, $\varepsilon_S \perp \eta_{n_p}, \eta_{n_p+1}, ..., \eta_n$. This implies that ε_S is normally distributed with a variance σ_S and is independent from θ. I assume the noisy term in the public signal is orthogonal to the private values of dealer $n_p, n_p + 1, ..., n$. We consider the natural case that these dealers' private values do not affect the generation of the public signal.

With the existence of the public signal S, for dealer $i \in \{n_p, ..., n\}$, their demand depends on this public signal. The demand functions of $i \in \{n_p, .., n-1\}$ are

$$\big(Q^i_{i-1,i}(\eta_i, p_{i-1,i}, S), Q^i_{i,i+1}(\eta_i, p_{i-1,i}, p_{i,i+1}, S)\big).$$

The demand function of dealer n is

$$Q^n_{n-1,n}(\eta_n, p_{n-1,n}, S).$$

Let $\hat{s}_i$ denote the private signal dealer i learns from trading with dealer $i-1$. From Lemma 2, we have the result that for $i \in \{2, ..., n_p\}$,

$$\hat{s}_i = s_i = s_1 + \frac{\kappa_1}{\varphi}\eta_1 + \frac{\kappa_2}{\varphi}\eta_2 + ... + \frac{\kappa_{i-1}}{\varphi}\eta_{i-1},$$

as their trade takes place before the dissemination of the public signal, and the information diffusion until dealer n_p is not affected. By contrast, for dealer $i \in \{n_p + 1, ..., n-1\}$, $\hat{s}_i \neq s_i$ as the public signal changes the information diffusion process since the trade between dealer n_p and $n_p + 1$.

The following proposition characterizes the trading behavior of dealer i and $i+1$ for $i \in \{n_p, n_p + 1, ..., n-1\}$, and the price function on the link $(i, i+1)$.

Proposition 2. *In the Linear Bayesian Nash equilibrium of the trading game between dealer i and $i+1$ ($i \in \{n_p, n_p+1, ..., n-1\}$), dealer i's demand is*

$$Q^i_{i,i+1} = a^i_{i,i+1}\hat{s}_i + b^i_{i,i+1}p_{i,i+1} + c^i_{i,i+1}\eta_i + d^i_{i,i+1}S,$$

dealer $i+1$'s demand is

$$Q^{i+1}_{i,i+1} = b^{i+1}_{i,i+1}p_{i,i+1} + c^{i+1}_{i,i+1}\eta_{i+1} + d^{i+1}_{i,i+1}S.$$

Define

$$\hat{\kappa}_i \equiv \frac{\sigma^2_\eta}{Var(\theta|S) - Var(\theta|\hat{s}_i, S)},$$

which is the inverse of information asymmetry between dealer i and $i+1$ scaled by σ^2_η. The coefficients have closed-form solution

$$b^i_{i,i+1} = b^{i+1}_{i,i+1} + \beta = \beta\hat{\kappa}_i, \qquad c^i_{i,i+1} = c^{i+1}_{i,i+1} = -\beta\hat{\kappa}_i,$$
$$\frac{c^i_{i,i+1}}{a^i_{i,i+1}} = \frac{\hat{\kappa}_i}{\varphi}\Big(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)}\Big).$$

The price function is

$$p_{i,i+1} = -\frac{a^i_{i,i+1}}{2\hat{\kappa}_i\beta}\hat{s}_i - \frac{d^i_{i,i+1} + d^{i+1}_{i,i+1}}{2\hat{\kappa}_i\beta}S + \frac{\eta_i + \eta_{i+1}}{2}.$$

Proposition 2 shows that the equilibrium of the trading game with a public signal is very similar to the equilibrium without, but with the following two differences. First, dealers' demands and the price function in equilibrium depend on the public signal. Second, the public signal affects the information asymmetry between dealers. Third, the private information diffusion process is not only decided by the information asymmetry $\hat{\kappa}_i$, but also by a new term $\frac{Cov(\hat{s}_i,S)}{Var(S)}$.

For dealer n_p and $n_p + 1$, their information asymmetry in the game with public signal, $Var(\theta|S) - Var(\theta|s_{n_p}, S)$, is different from that in the game without public signal, $Var(\theta) - Var(\theta|s_{n_p})$, as both of them learn from the public signal. Dealer n_p could learn a greater or smaller extent from S, $Var(\theta|s_{n_p}) - Var(\theta|s_{n_p}, S)$, than dealer $n_p + 1$, $Var(\theta) - Var(\theta|S)$.

For dealer i and $i+1$ ($i \in \{n_p+1, ..., n-1\}$), their information asymmetry in the game with public signal, $Var(\theta|S) - Var(\theta|\hat{s}_i, S)$, is different from that in the game without public signal, $Var(\theta) - Var(\theta|\hat{s}_i)$, as both of them learn from the public signal, and also the private signals learned by dealer i, $\hat{s}_i$ and s_i, are different in these two cases. Dealer i could learn a greater or smaller extent, $Var(\theta|s_i) - Var(\theta|\hat{s}_i, S)$, than dealer $i + 1$, $Var(\theta) - Var(\theta|S)$.

The following lemma shows the information diffusion process after the release of the public information.

Lemma 5. *In equilibrium, dealer $i + 1$ learns $\hat{s}_i$ from trading with dealer i ($i \in \{n_p, n_p + 1, ..., n - 1\}$),*

$$s_{i+1} = s_{n_p} + \frac{\hat{\kappa}_{n_p}}{\varphi}(1 - \frac{Cov(\hat{s}_{n_p}, S)}{Var(S)})\eta_{n_p} + ... + \frac{\hat{\kappa}_i}{\varphi}(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)})\eta_i,$$
$$\hat{\kappa}_{i+1} = \hat{\kappa}_i + \hat{\kappa}_i^2.$$

Using Lemma 2 and Lemma 5, we immediately have the following corollary that characterizes the necessary and sufficient condition for the effect of the public signal on the information asymmetry between dealer i and $i + 1$ ($i \in \{n_p, n_p + 1, ..., n - 1\}$).

Corollary 1. *Comparing the effect of the public signal on the information asymmetry between dealer i and $i+1$ for $i \in \{n_p+1, ..., n-1\}$,*

$$Var(\theta|S) - Var(\theta|\hat{s}_i, S) > Var(\theta) - Var(\theta|s_i)$$

$$\textit{iff} \quad Var(\theta|S) - Var(\theta|s_{n_p}, S) > Var(\theta) - Var(\theta|s_{n_p}).$$

If and only if the degree of information asymmetry between dealer n_p and n_p+1 becomes higher due to the existence of the public signal, then for any dealer $i \in \{n_p+1, ..., n-1\}$, the degree of information asymmetry between dealer i and $i+1$ becomes higher.

Corollary 1 shows that, to determine whether the introduction of a public signal mitigates or worsens adverse selection, it is sufficient to focus on the link immediately following the information release.

Now using the results in Proposition 2, I characterize dealers' surplus from the asset reallocation, as shown in the following lemma.

Lemma 6. *In equilibrium, the surplus from asset reallocation between dealer i and $i+1$ ($i \in \{n_p, n_p+1, ..., n-1\}$) is*

$$\mathbb{E}[(Q_{i,i+1}^{i+1}(\eta_{i+1} - \eta_i)] = -\beta\sigma_\eta^2\hat{\kappa}_i = \frac{-\beta\sigma_\eta^4}{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}.$$

Compared with its counterpart in the game without public signal

$$\mathbb{E}[(Q_{i,i+1}^{i+1}(\eta_{i+1} - \eta_i)] = \frac{-\beta\sigma_\eta^4}{Var(\theta) - Var(\theta|s_i)},$$

we can see whether the existence of a public signal would increase or reduce the surplus from asset reallocation between dealers depends on whether the degree of information asymmetry between dealers becomes higher or lower. Equivalently, it depends on whether or not dealer n_p can learn more from the public signal than dealer n_p+1.

Now I study two different information structures of the public signal. I first show the effect of transparency on dealers' trading surplus when the public signal is exogenous. Then I show the effect of transparency on dealers' trading surplus after endogenizing the public signal as the disseminated price of TRACE. I highlight the different implications in these two scenarios.

4.2 Exogenous Public Signal

I assume the public signal is exogenous, in the sense that the noisy term in S is independent from the noisy term in s_{n_p},

$$\varepsilon_S \perp (\varepsilon + \frac{\kappa_1}{\varphi}\eta_1 + \frac{\kappa_2}{\varphi}\eta_2 + ... + \frac{\kappa_{n_p-1}}{\varphi}\eta_{n_p-1}).$$

It can be shown that dealer $n_p + 1$ will learn more from the exogenous public signal than dealer n_p. Using Corollary 1, we have the result that the degree of information asymmetry between dealer i and $i+1$ becomes lower for $i \in \{n_p, n_p + 1, ..., n-1\}$, as shown in the following lemma.

Lemma 7. *If the public signal is exogenous, then in equilibrium, the degree of information asymmetry between dealer i and $i+1$ ($i \in \{n_p, n_p + 1, ..., n-1\}$) is lower than the case without a public signal,*

$$Var(\theta|S) - Var(\theta|\hat{s}_i, S) < Var(\theta) - Var(\theta|s_i).$$

The existence of the exogenous public signal would affect dealers' profits from clients, while it can be shown that the extent to which that is affected is modest. In contrast, the extent to which dealers' surplus from asset reallocation is affected is dramatic if the degree of information asymmetry becomes relatively low.

Lemma 8.

$$\lim_{\sigma_p \to 0} \mathbb{E}[(Q_{i,i+1}^{i+1}(\eta_{i+1} - \eta_i)] = \lim_{\sigma_p \to 0} \frac{-\beta\sigma_\eta^4}{Var(\theta|S) - Var(\theta|\hat{s}_i, S)} = \infty,$$

$$\lim_{\sigma_p \to 0} \mathbb{E}[\beta p_{i,i+1}(p_{i,i+1} - \theta - \eta_i)] \quad \textit{is bounded.}$$

If the public signal is extremely precise, the information asymmetry goes away, and the surplus from asset reallocation between dealer $i, i+1$ will explode, while dealers' profit from serving clients is bounded. Thus the profit of dealer i and $i+1$ from link $(i, i+1)$ is larger with a precise public signal than without a public signal.

This result is consistent with the conventional wisdom. The existence of the public signal reduces the information asymmetry, makes the adverse selection less severe, and thus increases the trading surplus in equilibrium by facilitating dealers' trade.

4.3 Endogenous Public Signal

In practice, TRACE collects the price of trades that have taken place and then disseminates the historical prices to the public. Thus the disseminated prices from the upstream trades are not exogenous public signals, because firstly, the prices of the upstream trades and the prices of the downstream trades could share the same information source (in my model, dealer 1's private signal s_1); secondly, they share the noisy terms that arise from upstream dealers' idiosyncratic trading needs (in my model, for example, dealer 1's private value η_1 enters the noisy terms of the prices in all the links.)

I assume the price of the first trade, the trading price between dealer 1 and 2, is the public signal,

$$S = \frac{2\kappa_1}{\varphi} p_{1,2} = s_1 + \frac{\kappa_1}{\varphi}(\eta_1 + \eta_2).$$

I make a technical assumption that $n_p \geq 3$. Namely, $p_{1,2}$ is observable after the trade between dealer 2 and 3 has finished. This assumption ensures that dealer 2 does not have an incentive to manipulate the public signal $p_{1,2}$ to profit from the trade with dealer 3.[3]

The following result shows that in markets with a relatively low degree of information asymmetry, the public signal would increase the degree of information asymmetry.

Proposition 3. *Compare the information asymmetry between dealer i and $i+1$ ($i \in \{n_p, ..., n-1\}$) with and without public signal, there is a threshold κ^* such that,*

$$Var(\theta|S) - Var(\theta|\hat{s}_i, S) > Var(\theta) - Var(\theta|s_i) \quad \text{if } \kappa_1 > \kappa^*.$$

If $n_p = 3$, $\kappa^* = 1$ is necessary and sufficient.

As shown before, for dealer i and $i+1$ ($i \in \{n_p, n_p+1, ..., n-1\}$), whether their information asymmetry in the game with a public signal, $Var(\theta|S) - Var(\theta|\hat{s}_i, S)$, is larger or smaller that in the game without a public signal, $Var(\theta) - Var(\theta|\hat{s}_i)$, depends on whether the information asymmetry between dealer n_p and $n_p + 1$, $Var(\theta|S) - Var(\theta|\hat{s}_{n_p}, S)$, is larger or smaller that in the game without a public signal, $Var(\theta) - Var(\theta|\hat{s}_{n_p})$. That is

[3] It can be proved that if we compare the cases that $p_{1,2}$ is observable or not for dealer 3 before dealer 3 trades with dealer 2, dealer 2 will behave differently in equilibrium. It is an interesting question how the price transparency affects price formation and information diffusion by incentivizing dealers to manipulate prices, though it is beyond the scope of this paper.

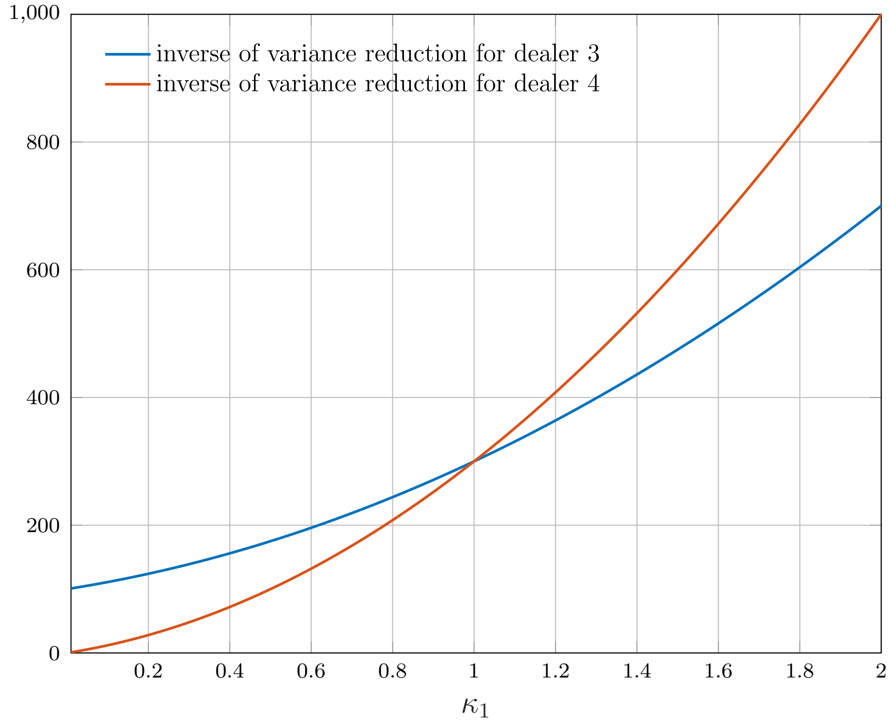

Figure 2: $\frac{1}{Var(\theta|s_3)-Var(\theta|s_3,S)}$ and $\frac{1}{Var(\theta)-Var(\theta|S)}$
Parameters: $n_p = 3, \sigma_\theta = 1, \sigma_\eta = 0.1$

equivalent to whether dealer n_p learns a greater or smaller extent from S, $Var(\theta|s_{n_p}) - Var(\theta|s_{n_p}, S)$, than dealer $n_p + 1$, $Var(\theta) - Var(\theta|S)$.

Figure 2 plots the inverse of variance reduction for dealer 3 and 4 due to the existence of the public signal when $n_p = 3$.

To see the intuition for this result, let us look at the case $n_p = 3$. In equilibrium, dealer 3's private signal is $s_3 = s_1 + \frac{\kappa_1}{\varphi}\eta_1 + \frac{\kappa_2}{\varphi}\eta_2$, and the public signal is $S = s_1 + \frac{\kappa_1}{\varphi}(\eta_1 + \eta_2)$. The key difference between the endogenous and the exogenous signal is that the endogenous signal is both about the asset payoff and previous dealers' private values, which allows the more informed dealer to filter its private signal more effectively. Intuitively, dealer $n_p = 3$ already has a signal about the private values of dealer 1 and 2. The endogenous public signal provides an additional signal that can make dealer 3 fully informed about η_2 and, by implication, fully informed about $s_2 = s_1 + \frac{\kappa_1}{\varphi}\eta_1$. But dealer 4 only learns S. When k_1 is relatively large, $\frac{\kappa_1}{\varphi}\eta_2$, the difference between s_2 and S that stems from the private value of dealer 2, becomes more noisy, and would increase the degree of information asymmetry between dealer 3 and 4.

To further see this, let us do a thought experiment by assuming the public signal is

$s_1 + \frac{\kappa_1}{\varphi}\eta_1 + \frac{\kappa^*}{\varphi}\eta_2$. Dealer 3's private signal is the same as before. Then from this public signal and $s_3 = s_1 + \frac{\kappa_1}{\varphi}\eta_1 + \frac{\kappa_2}{\varphi}\eta_2$, dealer 3 still learns the signal $s_2 = s_1 + \frac{\kappa_1}{\varphi}\eta_1$, while dealer 4 learns the signal $s_1 + \frac{\kappa_1}{\varphi}\eta_1 + \frac{\kappa^*}{\varphi}\eta_2$. We can see the value of κ^* that governs how pronounced the private value of dealer 2 is in the public signal, would decide whether the public signal would increase or reduce the degree of information asymmetry between dealer 3 and 4. Considering κ^* approaches 0, then the information asymmetry between dealer 3 and 4 is almost removed; thus, the degree of information asymmetry becomes smaller. Suppose κ^* goes to infinity, then dealer 4 basically can not learn anything from the public signal (the information asymmetry between 3 and 4 approaches the information asymmetry between dealer 2 and 3); thus, the degree of information asymmetry becomes larger. [4]

From this thought experiment, we can see the effect of the public signal on the degree of information asymmetry is decided by the magnitude of κ^* or how significantly dealer 2's private value affects the public signal. From the price function shown in Proposition 1, we have in equilibrium $\kappa^* = \kappa_1$. The degree of information asymmetry between dealer 1 and 2 decides how aggressively dealer 2 trades on its private value, and thus decides how significantly its private value affects the public signal, which decides whether the public signal would increase or reduce the degree of information asymmetry between the subsequent dealers.

The existence of the endogenous public signal would affect dealers' profits from serving the clients, while it can be shown that qualitatively, that effect is dominated by the effect on dealers' surplus from asset reallocation if the degree of information asymmetry is relatively low, as shown by the following proposition.

Proposition 4. *Trading profit of dealer i and $i+1$ ($i \in \{n_p, ..., n-1\}$)*

$$\mathbb{E}(\pi^i_{i,i+1}) + \mathbb{E}(\pi^{i+1}_{i,i+1}) = \mathbb{E}[(Q^{i+1}_{i,i+1}(\eta_{i+1} - \eta_i)] + \mathbb{E}[\beta p_{i,i+1}(p_{i,i+1} - \theta - \eta_i)]$$

is smaller with a public signal than without a public signal if κ_1 is relatively large.

[4]Formally, the change of information asymmetry is

$$Var(\theta|s_1 + \frac{\kappa_1}{\varphi}\eta_1 + \frac{\kappa^*}{\varphi}\eta_2) - Var(\theta|s_3, s_1 + \frac{\kappa_1}{\varphi}\eta_1 + \frac{\kappa^*}{\varphi}\eta_2) - (Var(\theta) - Var(\theta|s_3))$$

$$= -\frac{\sigma_\theta^2}{\kappa_1 + \kappa_1^2 + \kappa_1^{*2}}\varphi + \frac{\sigma_\theta^2}{\kappa_2}\varphi - \frac{\sigma_\theta^2}{\kappa_3}\varphi = \sigma_\theta^2\varphi(\frac{1}{\kappa_2} - \frac{1}{\kappa_3} - \frac{1}{\kappa_1 + \kappa_1^2 + \kappa_1^{*2}}) = \sigma_\theta^2\varphi\frac{\kappa_2(\kappa^{*2} - 1)}{(\kappa_2 + \kappa_2^2)(\kappa_1 + \kappa_1^2 + \kappa^{*2})}.$$

Figure 3 plots when $n_p = 3$, dealer 3 and dealer 4's trading profits from link (3,4), and dealer 4 and dealer 5's trading profits from link (4,5). The blue line plots the profits when the market is opaque, and the orange line plots the profits when the trading price between dealer 1 and 2, $p_{1,2}$, is public.

From the simulation results, we can see when κ_1 is above a threshold that is very close to 1 — which is a threshold for the information asymmetry to be larger or smaller — dealers' trading profits from link (3,4) or link (4,5) are higher in the case that the market is opaque. The observation from Figure 3 that these two thresholds are almost the same holds for all the parameters that I have used for the simulation and for any $n_p \geq 3$. That indicates that for the wide range of κ_1, the effect on dealers' information asymmetry, and thus on dealers' trading surplus from asset reallocation, plays a dominant role in affecting dealers' trading profit.[5]

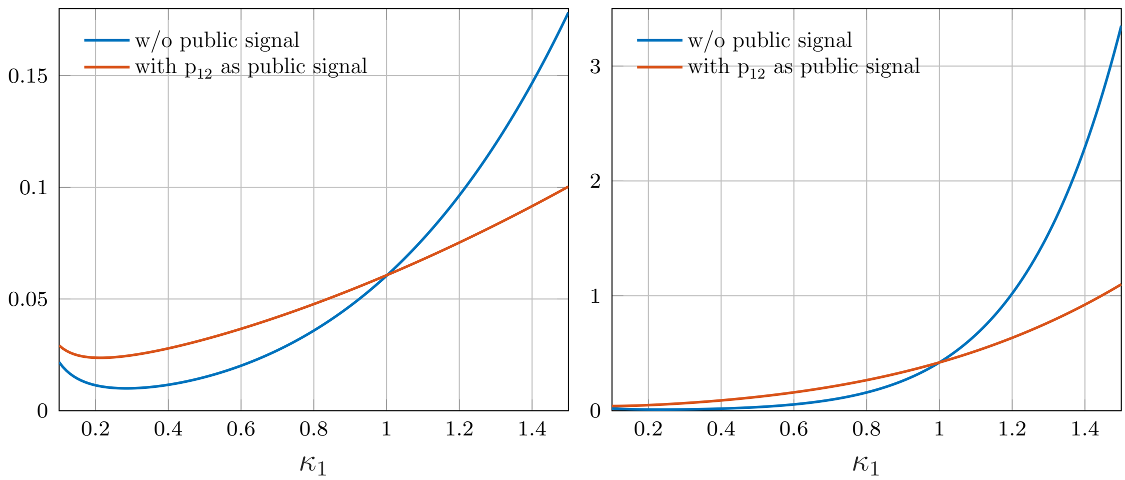

Figure 3

$\mathbb{E}(\pi^3_{3,4}) + \mathbb{E}(\pi^4_{3,4})$, $\mathbb{E}(\pi^4_{4,5}) + \mathbb{E}(\pi^5_{4,5})$

Parameters: $n_p = 3$, $\beta = -1$, $\sigma_\theta = 1, \sigma_\eta = 0.1$

5 The Network Formation Game

In this section, I propose a network formation model to study the effect of TRACE on the inter-dealer network structure. The network formation stage is before the trading stage.

[5] When κ_1 is very close to 0, sometimes dealers' trading profit is higher in the case that the market is opaque.

In this model, the dealer with private information about the asset payoff bargains with the second dealer over how to split the link formation cost; the second dealer who forms the link bargains with the third dealer over how to split the link formation cost, etc. In equilibrium, the surplus of each formed link would spread over all the dealers before this link and affect their network formation decision. Thus the effect of TRACE on downstream dealers' trading profit would affect the upstream dealers' link formation decision. The lower trading profits of downstream dealers could hurt the the formation of the whole network.

I show that, in a market with less information asymmetry, the inter-dealer trading frequency becomes lower after TRACE. The network formation game also implies that TRACE could reduce the trading frequency of the upstream dealers as well, because they benefit less from the downstream dealers' trading profits if forming the links. The empirical implications are tested in Section 7 and leave to be done more empirical work in the future.

5.1 The Model Setup

There is a set of risk-neutral dealers $\{1, 2, .., \bar{n}\}$. The link formation cost C is drawn from distribution $F(C)$. Given the realization of C, the link formation problem between dealer 1 and 2 is modeled as Nash's two-person bargaining problem with fixed disagreement payoffs, which are 0; if dealer 1 and 2 agree to form the link, we proceed to the link formation problem between dealer 2 and 3, etc; between two consecutive dealers, whether the agreement is reached and how the cost C is split are determined according to the symmetric Nash bargaining solution. For $i \in \{2, .., \bar{n}\}$, $l_{i-1,i} = 1$ denotes the link is formed between dealer $i-1$ and dealer i; $l_{i-1,i} = 0$ otherwise. $(i-1, i)$ denotes the formed link between dealer $i-1$ and dealer i.

$C^{i-1}_{i-1,i}$ and $C^{i}_{i-1,i}$ denote the costs paid by dealer $i-1$ and dealer i respectively when the link is formed between them. $C^{i-1}_{i-1,i} + C^{i}_{i-1,i} = C$. $\mathbb{E}(\pi^{i}_{i-1,i})$ and $\mathbb{E}(\pi^{i}_{i,i+1})$ denotes the expected payoff of dealer i from trading on the link $(i-1, i)$ and link $(i, i+1)$ respectively.

Dealer 1's payoff

$$U_1(l_{1,2}, C^1_{1,2}) = l_{1,2}(\mathbb{E}(\pi^1_{1,2}) - C^1_{1,2}).$$

Dealer 2's payoff

$$U_2(l_{1,2}, C^2_{1,2}, l_{2,3}, C^2_{2,3}) = l_{1,2}\big(\mathbb{E}(\pi^2_{1,2}) - C^2_{1,2} + l_{2,3}(\mathbb{E}(\pi^2_{2,3}) - C^2_{2,3})\big).$$

For $i \in \{2, ..., \bar{n}-1\}$, dealer i's payoff is

$$U_i(l_{i-1,i}, C^i_{i-1,i}, l_{i,i+1}, C^i_{i,i+1}) = l_{i-1,i}\big(\mathbb{E}(\pi^i_{i-1,i}) - C^i_{i-1,i} + l_{i,i+1}(\mathbb{E}(\pi^i_{i,i+1}) - C^i_{i,i+1})\big).$$

For dealer $\bar{n}$, its payoff is

$$U_{\bar{n}}(l_{\bar{n}-1,\bar{n}}, C^{\bar{n}}_{\bar{n}-1,\bar{n}}) = l_{\bar{n}-1,\bar{n}}\big(\mathbb{E}(\pi^{\bar{n}}_{\bar{n}-1,\bar{n}}) - C^{\bar{n}}_{\bar{n}-1,\bar{n}}\big).$$

Whether the link is formed and how the cost is split are determined according to the symmetric Nash bargaining solution.

In the main text of the paper, I restrict the network to a line or chain. This framework can be extended to study the formation of any tree network, but all the qualitative results will be the same as what I show in the main text.[6] After the network is formed, dealers' private values and dealer 1's private signal s_1 are realized, and then the sequential trading takes place.[7]

5.2 Equilibrium Concept

Definition 2. *n denotes the number of dealers in network. $\big(n, (\hat{l}_{i-1,i})_{i\in\{2,..,n\}}, (\hat{C}^{i-1}_{i-1,i})_{i\in\{2,..,n\}}\big)$ is an equilibrium if and only if*

(1) $\hat{C}^{i-1}_{i-1,i}$ is the symmetric Nash bargaining solution

$$\hat{C}^{i-1}_{i-1,i} = \arg\max_{C^{i-1}_{i-1,i}} \Big(\mathbb{E}(\pi^{i-1}_{i-1,i}) - C^{i-1}_{i-1,i}\Big)^{\frac{1}{2}},$$

$$\Big(\mathbb{E}(\pi^i_{i-1,i}) - (C - C^{i-1}_{i-1,i}) + \mathbf{1}(i<n)\big((\mathbb{E}(\pi^i_{i,i+1}) - \hat{C}^i_{i,i+1}\big)\Big)^{\frac{1}{2}},$$

where $\big((\mathbb{E}(\pi^i_{i,i+1}) - \hat{C}^i_{i,i+1}\big)$ is the continuation value for $i<n$. Thus $\hat{l}_{i-1,i} = 1$ if and only if $\mathbb{E}(\pi^{i-1}_{i-1,i}) \geq C^{i-1}_{i-1,i}$ and $\hat{l}_{j-1,j} = 1$ for all $j \in \{2, ..., i-1\}$.

(2) Let $\hat{l}_{0,1} = 1$. n is the maximal integer in $\{1, ..., \bar{n}\}$ such that $\hat{l}_{i-1,i} = 1$ for any $i \in \{1, ..., n\}$.

[6] For example, I am able to extend the network formation game, such that the dealer that has formed the link can contact and negotiate with each of a random number of other dealers, and decide whether to form links with them. The details are available upon request.

[7] This setup of timing is in the spirit of security design literature, for example, DeMarzo and Duffie (1999), such that I do not need to deal with the thorny signaling problem during the network formation process. The network structure in equilibrium only depends on the primitives of the model, rather than the realization of the random variables.

5.3 Equilibrium of the Network Formation Game

Conditional on all the previous links being formed, whether the last link can be formed or not just depends on whether the surplus of trading on this link, $\mathbb{E}(\pi^{n-1}_{n-1,n}) + \mathbb{E}(\pi^{n}_{n-1,n})$, is larger than the link formation cost.

Conditional on all the links before the penultimate link being formed, $\mathbb{E}(\pi^{i-1}_{i-1,i}) + \mathbb{E}(\pi^{i}_{i-1,i}) - C \geq 0$ does not ensure the penultimate link can be formed. For the penultimate link in the chain, whether it can be formed or not depends on the surplus of trading on this link and the net benefit of dealer $n-1$ from the last link, $\mathbb{E}(\pi^{i}_{i,i+1}) - \hat{C}^{i}_{i,i+1}$, which is affected by the surplus of the last link. Solving the symmetric Nash Bargaining problem for the penultimate link and substituting in the symmetric Nash bargaining solution of the last link, we have the penultimate link able to be formed if and only if

$$\frac{\mathbb{E}(\pi^{n-2}_{n-2,n-1}) + \mathbb{E}(\pi^{n-1}_{n-2,n-1}) + \frac{1}{2}\big(\mathbb{E}(\pi^{n-1}_{n-1,n} + \mathbb{E}(\pi^{n}_{n-1,n})\big)}{1+\frac{1}{2}} \geq C.$$

Repeating this procedure for all the previous links until the first link, I can get the cost threshold for each length of the network. If the realized cost is below that threshold, the network with that length can be formed in equilibrium. The following algorithm formalizes this procedure.

Algorithm that solves the network formation game

Considering $n = \bar{n}$.

Link $(\bar{n}-1, \bar{n})$ is formed if and only if $C \leq \tilde{C}_{\bar{n}-1,\bar{n}}(\bar{n})$, where

$$\tilde{C}_{\bar{n}-1,\bar{n}}(\bar{n}) \equiv \mathbb{E}(\pi^{\bar{n}-1}_{\bar{n}-1,\bar{n}}) + \mathbb{E}(\pi^{\bar{n}}_{\bar{n}-1,\bar{n}}).$$

Link $(i-1, i)$ for $i \in \{2, \bar{n}\}$ is formed if and only if $C \leq \tilde{C}_{i-1,i}(\bar{n})$, where

$$\tilde{C}_{i-1,i}(\bar{n}) \equiv \frac{\mathbb{E}(\pi^{i-1}_{i-1,i}) + \mathbb{E}(\pi^{i}_{i-1,i}) + \frac{1}{2}\big(\mathbb{E}(\pi^{i}_{i,i+1}) + \mathbb{E}(\pi^{i+1}_{i,i+1})\big) + \ldots \frac{1}{2^{\bar{n}-i}}\big(\mathbb{E}(\pi^{\bar{n}-1}_{\bar{n}-1,\bar{n}}) + \mathbb{E}(\pi^{\bar{n}}_{\bar{n}-1,\bar{n}})\big)}{1+\frac{1}{2}+\ldots\frac{1}{2^{\bar{n}-i}}}.$$

Define $C^*(\bar{n}) \equiv \min\{\tilde{C}_{1,2}(\bar{n}), ..., \tilde{C}_{\bar{n}-1,\bar{n}}(\bar{n})\}$.

In equilibrium the trading network has $\bar{n}$ dealers if and only if $C \leq C^*(\bar{n})$.

If $C > C^*(\bar{n})$, we let $n = \bar{n}-1$, and characterize $C^*(\bar{n}-1) \equiv \min\{\tilde{C}_{1,2}(\bar{n}-1), ..., \tilde{C}_{\bar{n}-2,\bar{n}-1}(\bar{n}-1)\}$.

In equilibrium the network has $\bar{n}-1$ dealers if and only if $C^*(\bar{n}) < C \leq C^*(\bar{n}-1)$.

If $C > \max\{C^*(\bar{n}), C^*(\bar{n}-1)\}$, we let $n = \bar{n}-2$, and characterize $C^*(\bar{n}-2) \equiv \min\{\tilde{C}_{1,2}(\bar{n}-2), ..., \tilde{C}_{\bar{n}-3,\bar{n}-2}(\bar{n}-2)\}$.

In equilibrium the network has $\bar{n}-2$ dealers if and only if $\max\{C^*(\bar{n}), C^*(\bar{n}-1)\} < C \leq C^*(\bar{n}-2)$.

...

The link between dealer $i-1$ and i is formed in equilibrium, if and only if the network formed in equilibrium has at least i dealers. Thus in equilibrium $\hat{l}_{i-1,i} = 1$ for $i \in \{2, 3, ..., \bar{n}\}$, if and only if $C \leq C^*_{i-1,i}(\kappa_1)$, where

$$C^*_{i-1,i}(\kappa_1) \equiv \max\{C^*(i), ..., C^*(\bar{n})\}.$$

I denote $C^*_{i-1,i,np}(\kappa_1)$ to be the threshold in the game without a public signal, and $C^*_{i-1,i,p}(\kappa_1)$ to be the threshold in the game with $p_{1,2}$ as the public signal.

Proposition 5. *In equilibrium of the network formation game, when κ_1 is relatively large,*

(1) for $i \in \{n_p+1, ..., \bar{n}\}$, $C^*_{i-1,i,np}(\kappa_1) > C^*_{i-1,i,p}(\kappa_1)$;

(2) for $i \in \{2, ..., n_p\}$, $C^*_{i-1,i,np}(\kappa_1) \geq C^*_{i-1,i,p}(\kappa_1)$.

Proposition 5 is directly implied by Proposition 4. When κ_1 is relatively large, for any two consecutive dealers that are affected by the public signal, due to the larger information asymmetry between them, they have lower trading profit. That lowers the threshold of cost for forming the link between them.

It also implies that TRACE could reduce the trading frequency of the upstream dealers for whom trading profits are not affected by the public signal. That is because they benefit less from the downstream dealers' trading profits if they form the link to initiate the trade. Figure 4 plots $C^*_{1,2,np}(\kappa_1)$ in the game without a public signal and $C^*_{1,2,p}(\kappa_1)$ in the game with $p_{1,2}$ as the endogenous public signal. In markets with relatively large κ_1, the public signal reduces the cost threshold of the first link's formation, and thus reduces the ex-ante probability of any inter-dealer trade.

6 Empirical Analysis

In this section, I study the effect of TRACE on corporate bond markets, by utilizing the Academic Corporate Bond TRACE data and the Mergent FISD database. The argument is that if a bond is very actively traded, then although investors are more likely to know more about them, they are also more likely to be asymmetrically informed; vice versa, if

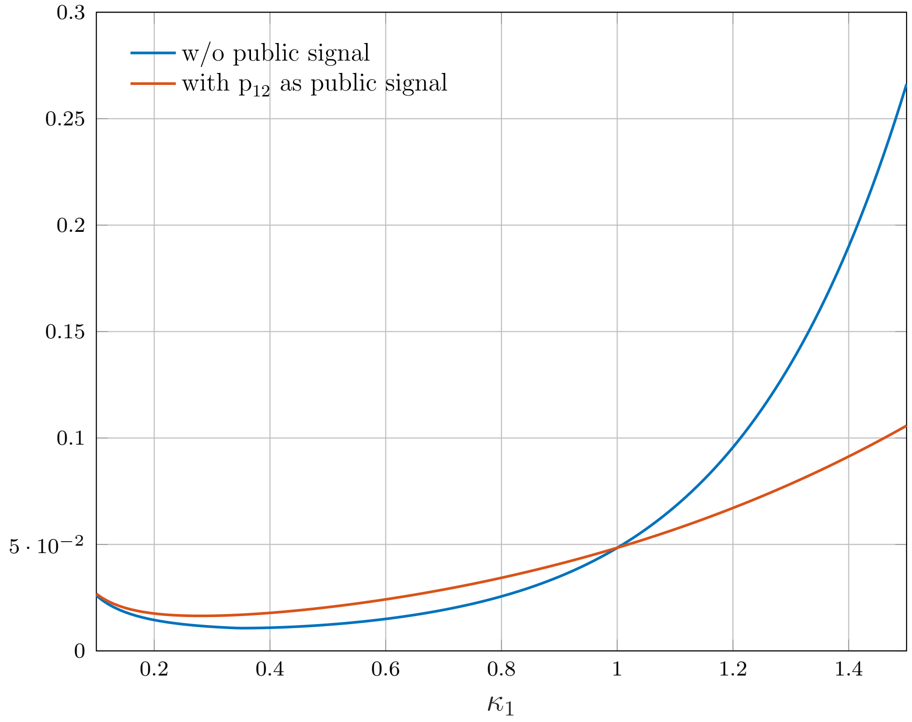

Figure 4: $C^*_{1,2,np}(\kappa_1)$ and $C^*_{1,2,p}(\kappa_1)$

Parameters: $\beta = -1$, $\sigma_\theta = 1, \sigma_\eta = 0.1$, $n = 5$, $n_p = 3$

a bond is thinly traded, then investors are more likely to be symmetrically uninformed. So the model predicts that TRACE would have negative effects on inter-dealer trading activity for thinly traded bonds.

The regression results indicate that TRACE has a significant negative effect on the inter-dealer trading frequency for thinly traded bonds. It is also a warning for the widespread implementation of TRACE, as many of the assets that are newly subject to transparency are also thinly traded.

The main difference between my empirical work and Asquith, Covert and Pathak (2019) is that, the trading activity in their paper is the aggregate trading activity (including the trade between dealer and customer and inter-dealer trade), while mine is the inter-dealer trading activity (and further separate inter-dealer trade into principle inter-dealer trade and agency inter-dealer trade).

6.1 Data Description

The data source for corporate bond trading is the Academic Corporate Bond TRACE data, purchased from FINRA. During the time period, July 1, 2002 until December 31, 2005, there are 29,064,905 unique trade reports on 37,026 different CUSIPs in the Academic TRACE dataset.

The Mergent FISD database is my source for bond characteristics such as issue size, credit ratings, maturity, etc., which I add to the Academic TRACE dataset. The Mergent FISD database I use include all the bonds with an offering date between January of 1950 and January of 2010.

6.2 Steps from the Academic TRACE File to the Cleaned Academic TRACE Sample

There are a number of reporting errors in this self-reported data. The appendix describes the steps I take to convert the Academic TRACE File to the Cleaned Academic TRACE Sample. The references for the cleaning procedure are Asquith, Covert and Pathak (2019) and Dick-Nielsen (2014). Table 1 reports the number of bonds and trade reports after each step.

6.3 Steps from FINRA's Phase Listings to the Cleaned Phase Sample

Dissemination took place in Phases over two-and-a-half years. FINRA's main criteria for a bond's dissemination Phase are the bond issue size and credit rating. Actively traded, investment grade bonds became transparent before thinly traded, high-yield bonds. Phase 1 of TRACE was implemented on July 1, 2002. Phase 2 of TRACE was implemented on March 3, 2003. Phase 3A of TRACE was implemented on October 1, 2004. Phase 3B of TRACE was implemented on February 7, 2005.

I begin with a list of all Phase 1, 2, 3A, and 3B bonds. There are 16,854 bonds in this list, of which 15,769 exist in the Cleaned Academic TRACE sample. They have 3,526,543 trades during our sample period.

In addition to the four Phases that correspond to the FINRA dissemination dates, FINRA also maintained two other lists of bonds, which we call the FINRA50 and the FINRA120[8]. The FINRA50 represent 50 Non-Investment Grade (High-Yield) securities

[8]Dissemination began on April 14, 2003 for a group of 120 Investment-Grade securities rated BBB. These BBB bonds are denoted as the FINRA120.

	Remaining CUSIPs	Remaining trade reports
Source: Academic TRACE	37,026	29,064,905
Eliminate trade btw dealer and customer	32,795	11,900,080
Eliminate bonds based on characteristics		
Bonds unmatched to FISD by CUSIP	31,682	11,800,641
Convertible bonds and exchangeable bonds	29,872	11,005,869
SEC Rule 144a bonds	26,965	10,704,976
Bonds with 0 or very small issue size	26,729	10,687,352
Eliminate trade because of self-reported errors		
Same day corrections and cancellations	26,647	10,423,151
Reversals: ten-way match	26,581	10,121,712
Reversals: nine-way match	26,578	10,113,094
Reversals: nine-way match (price rounding to 0.01)	26,578	10,112,431
Eliminate double report	25,239	5,155,102
Address trade splitting	25,239	4,889,149
Eliminate trade reports with price or volume issues		
Prices that are vastly out of line	25,239	4,885,676
Prices/volumes below 0.01% or above 99.99%	25,181	4,883,688
Eliminate trade that is under special circumstances, etc.	25,101	4,865,045

Table 1: Steps from Academic TRACE File to Cleaned Academic TRACE Sample

disseminated under the Fixed Income Pricing System (FIPS)[9]. This list of 50 bonds

[9]FIPS started in April 1994. It reported transactions information on approximately 50 high-yield bonds.

	CUSIPs	Trade reports
Sample 1: Cleaned Academic TRACE Sample	25,101	4,865,045
Sample 2: FINRA list of Phase 1-3B bonds	16,854	-
Sample 3: Bonds in both Sample 1 and 2	15,769	3,526,557
Bonds in Sample 3 but not in FINRA50	15,761	3,517,632
(**Cleaned Phase Sample**)		
Phase 1	370	1,031,409
Phase 2	2,348	668,300
Phase 3A	10,492	1,613,741
Phase 3B	2,551	204,182

Table 2: Steps from FINRA's Phase Listings to the Cleaned Phase Sample

changes over time with bonds both entering and exiting. I eliminate any bonds that also exist in the FINRA50. There are 3,517,618 unique trade reports (phase 1: 1,031,396, phase 2: 668,300, phase 3A: 1,613,748, phase 3B: 204,174) and 15,781 different CUSIPs (phase 1: 370, phase 2: 2,348, phase 3A: 10,492, phase 3B: 2,551) left.

6.4 Key Statistics

Different from other bond characteristics, a bond usually has multiple credit ratings that are specific to rating date. I assign credit ratings that are rated since July 2002. Data on credit ratings are from FISD. FISD includes ratings from S&P, Moody's, Fitch and Duff and Phelps. I first transform Moody's rating to be consistent with others. For the phase 2 (3A, 3B) exercise, for each bond in the credit rating data, I keep the observation with the rating date that is after and closest to July 2002.[10]

[10] As an alternative, I could use the bond's credit rating that was established after its phase began. I do not use that, because many bonds, especially Phase 3B bonds, were not rated anymore after the start of Phase 3B. For example, for the 2551 Phase 3B bonds in my sample, there are only 1126 bonds that

From Table 3, we can see Phase 3B bonds have significant lower credit ratings than bonds in other Phases; Phase 3A bonds have much smaller issue size.

	Phase 2	Phase 3A	Phase 3B
Number of bonds	2,348	10,492	2,551
Number of bonds Issue size ($M)			
mean	274	88	185
p5	100	1	1
p10	100	1	5
p25	150	3	100
median	237	12	150
p75	325	92	235
p90	500	300	350
p95	600	450	450
Credit rating			
mean	A+	A-	B+
p5	AAA	AAA	BBB
p10	AA	AA	BBB-
p25	AA-	A+	BB
median	A+	A-	BB-
p75	A	BBB+	B-
p90	A-	BBB-	CCC
p95	A-	BB+	CC
# of bonds with rating information	2,200	10,334	2,142

Table 3: Bond Issue Size and Credit Rating

For the regression in the next section, I construct three subsamples. The Phase 2 subsample keeps the transactions from December 3, 2002 to June 3, 2003; the Phase 3A subsample keeps the transactions from July 1, 2004 to the end of 2004; the Phase 3B subsample keeps the transactions from November 7, 2004 to May 7, 2005. For each

are rated after February 7, 2005, when TRACE was implemented for Phase 3B.

subsample, I fill in the bonds that are not observable within that subsample but are observable in other subsamples, and count their number of trades as 0; then I drop the bond if its offering day is after the policy implementation date or its maturity day is before the policy implementation date or its credit rating information is missing.

For each subsample, I calculate each bond's number of inter-dealer trade per day within 3 months before the policy implementation and 3 months after the policy implementation.[11] Table 4 shows the average number of trades per day for Phase 2 bonds in the Phase 2 subsample, Phase 3A bonds in the Phase 3A subsample, and Phase 3B bonds in the Phase 3B subsample. We can see Phase 3B bonds are much less frequently traded than Phase 2 and Phase 3A bonds.

	num of trades per day within 3 months before	num of trades per day within 3 months after
Phase 2 bonds	0.2479	0.2857
Phase 3A bonds	0.1289	0.1286
Phase 3B bonds	0.0486	0.0509

Table 4: Number of Overall Inter-dealer Trades per Day by Phase

In some trades, dealers may act as agent. In agency transactions, a dealer does intermediation by transferring the bond while not assuming any price risk, thus the agency transaction does not reflect the transfer of bond ownership or price risk. The principal inter-dealer trade is defined to be the trade between two dealers whose trading capacities are both principal. The agency inter-dealer trade is defined to be the trade between two dealers and at least one side's trading capacity is principal. I separate the over-all inter-

[11] If the bond's offering day is within 3 months before the policy implementation, its number of trades per day is the total number of trades before the policy implementation divided by the number of days between its offering day and the policy implementation. If the bond's maturity day is within 3 months after the policy implementation, its number of trades per day is the total number of trades after the policy implementation divided by the number of days between the policy implementation and its maturity day.

dealer trades into agency inter-dealer trades and principal inter-dealer trades, to study the separate effects of TRACE on them.

Table 5 reports each bond's number of trades per day for principal inter-dealer trade by phase. We can see for all the bonds, most of their inter-dealer trades are principal inter-dealer trades. In terms of principal inter-dealer trade, Phase 3B bonds are still much less frequently traded than Phase 2 and Phase 3A bonds. Table 6 reports each bond's number of trades per day for agency inter-dealer trade by phase. In terms of agency inter-dealer trade, Phase 3B bonds' trading frequency is similar to Phase 3A bonds, while Phase 3A and Phase 3B bonds are much less frequently traded than Phase 3 bonds.

	num of trades per day within 3 months before	num of trades per day within 3 months after
Phase 2 bonds	0.2139	0.2473
Phase 3A bonds	0.1158	0.1171
Phase 3B bonds	0.0397	0.0398

Table 5: Number of Principal Inter-dealer Trades per Day by Phase

	num of trades per day within 3 months before	num of trades per day within 3 months after
Phase 2 bonds	0.0394	0.0445
Phase 3A bonds	0.0167	0.0147
Phase 3B bonds	0.0124	0.0152

Table 6: Number of Agency Inter-dealer Trades per Day by Phase

6.5 Information Asymmetry for Bonds in Difference Phases

Using the result of this model, I derive a new measure that reflects κ_1 — the information asymmetry scaled by the variance of private values in opaque markets. The following lemma shows how this measure is constructed by using the price variance. The merit of this measure is that it identifies κ_1 even when σ_η^2 is heterogenous across markets. But it requires that σ_η^2 is the same for all the dealers that trade on different links for the same bond.

Lemma 9. *In equilibrium of the trading game without public signal,*

$$Var(p_{i,i+1}) = \frac{\sigma_\eta^2}{4\kappa_i} + \frac{\sigma_\eta^2}{2},$$

thus

$$\frac{Var(p_{23}) - Var(p_{34})}{Var(p_{12}) - Var(p_{23})} = \frac{\frac{1}{\kappa_2} - \frac{1}{\kappa_3}}{\frac{1}{\kappa_1} - \frac{1}{\kappa_2}} = \frac{1 + \kappa_1}{1 + \kappa_1 + \kappa_1^2}$$

is decreasing in κ_1.

For each bond, I identify the trade between dealer 1 and 2 if this trade is the first trade for both them within the past 1 month.

I identify the trade between dealer 2 and 3 if:

(1) this trade is the first trade for one dealer within the past 1 month;

(2) this trade is the second trade for the other dealer within the past 1 month whose first trade is identified as the trade between dealer 1 and 2.

I identify the trade between dealer 3 and 4 if:

(1) this trade is the first trade for one dealer within the past 1 month;

(2) this trade is the third trade for the other dealer within the past 1 month whose first trade is identified as the trade between dealer 2 and 3.

For each bond, I calculate $Var(p_{12}), Var(p_{23}), Var(p_{34})$, which are the variances of prices on the link between dealer 1 and 2, between dealer 2 and 3, between dealer 3 and 4, before the start of Phase 2 — March 3, 2003. As predicted by the model, if σ_η^2 is the same for all the dealers that trade on different links, we have $\frac{Var(p_{23})-Var(p_{34})}{Var(p_{12})-Var(p_{23})}$. Thus I only keep the bonds if its $Var(p_{12}) > Var(p_{23}) > Var(p_{34})$.

For each bond, I calculate $\frac{Var(p_{23})-Var(p_{34})}{Var(p_{12})-Var(p_{23})}$. The outliers are dropped. Then I calculate the average $\frac{Var(p_{23})-Var(p_{34})}{Var(p_{12})-Var(p_{23})}$ for each Phase. The results are 2.44 for Phase 2, 2.09 for

Phase 3A, 1.98 for Phase 3B. This is the evidence that for Phase 3B bonds, the degree of information asymmetry is systematically lower.

6.6 Difference-in-Differences Regression

The before-and-after comparisons in Table 4 do not establish that dissemination affected trading activity because there could be contemporaneous market-wide trends. I adjust for potential market trends by comparing the changes in the sample of newly disseminated bonds (the treated sample) to those that do not change dissemination status (the control sample) by estimating difference-in-differences models of the form:

$$y_{it} = \lambda_1 \text{Disseminate}_i + \lambda_2 \text{Post}_t + \lambda_3 \text{Disseminate}_i \times \text{Post}_t + \boldsymbol{\lambda}_4 \boldsymbol{X}_{it} + \varepsilon_{it}$$

where i denotes the bond, t denotes the time period (3 months before/after the dissemination), y_{it} is bond i's outcome (number of trades per day), Disseminate_i is an indicator for whether the bond changes dissemination status (i.e., is in the treated group), Post_t is an indicator for the trade outcomes after the dissemination, and $\boldsymbol{X}_{it}$ is a vector of bond i's characteristics (issue size and credit rating).

Any pre-existing difference between bonds that change dissemination status and those that do not are captured by λ_1. Any effects of dissemination that accrue to all bonds — that is, effects that are not limited to only bonds that change their dissemination status in the Phase — are absorbed by time effects λ_2. The coefficient of interest is λ_3, which estimates the direct effect of transparency on the outcome variable. The coefficient λ_3 reflects the change in trading outcomes for bonds that change dissemination status compared to the change in trading outcomes for bonds that do not change dissemination status. Estimates of λ_3 therefore, net out aggregate changes in bond trading outcomes.

The control bonds for Phase 2 are the disseminated bonds in Phase 1, and the non-disseminated bonds in Phase 3A and Phase 3B. For Phase 3A and Phase 3B, the control bonds are the disseminated bonds in Phase 1 and Phase 2. Phase 3A bonds are not a control for Phase 3B and vice versa because Phase 3A and Phase 3B occur just over four months apart, on October 1, 2004 and February 7, 2005, respectively. For λ_3 to provide unbiased estimates of the causal effect of transparency, I assume that the change over time in control bonds' behavior reveals what would have occurred to treated bonds if there had been no change in their dissemination status.[12]

[12]The other necessary assumptions are, first, that transparency and its consequences are not well

Table 7 reports the estimates of equation for overall inter-dealer trades of Phase 2 bonds, Phase 3A bonds and Phase 3B bonds. For Phase 2 bonds and Phase 3A bonds, TRACE post-trade transparency does not have significant effect, for Phase 3B bonds, it significantly reduces the number of trades. TRACE reduces the average number of overall inter-dealer trades for Phase 3B bonds by 0.09. This represents a 190% drop from the average level before dissemination.

	Phase 2 num of trades per day	Phase 3A num of trades per day	Phase 3B num of trades per day
Disseminate	-0.303***	-0.205***	-0.994***
	(-10.79)	(-5.63)	(-5.09)
Post	0.0250***	-0.0151*	0.0947***
	(4.72)	(-2.00)	(3.80)
Disseminate×Post	0.0128	0.0148	-0.0924***
	(1.57)	(1.86)	(-3.69)
N	19998	24946	8418

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 7: DID Regression Results for Overall Inter-dealer Trade

Table 8 reports the estimates of equation for principal inter-dealer trades. For Phase 2 bonds and Phase 3A bonds, TRACE post-trade transparency does not have a negative effect, whereas for Phase 3B bonds it significantly reduces the number of principal inter-dealer trades.

Table 9 reports the estimates of equation for agency inter-dealer trades. For Phase 2 bonds, TRACE post-trade transparency does not have a significant effect, whereas for

anticipated by market participants and thus the impacts on the trading activity would not appear before the actual change in dissemination status; and secondly, that there are no other changes simultaneous with the phase start date that affects the trading activity for those bonds changing dissemination status, as argued in Asquith, Covert and Pathak (2019).

	Phase 2 num of trades per day	Phase 3A num of trades per day	Phase 3B num of trades per day
Disseminate	-0.260***	-0.137***	-0.872***
	(-10.75)	(-4.29)	(-5.18)
Post	0.0213***	-0.0259***	0.0810***
	(4.21)	(-4.13)	(3.54)
Disseminate×Post	0.0121	0.0273***	-0.0809***
	(1.56)	(4.06)	(-3.53)
N	19882	24896	8314

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 8: DID Regression Results for Principal Inter-dealer Trade

Phase 3A and Phase 3B bonds it significantly reduces the number of agency inter-dealer trades. TRACE reduces the average number of agency inter-dealer trades for Phase 3A bonds by 0.0138. This represents a 83% drop from the average level before dissemination. TRACE reduces the average number of agency inter-dealer trades for Phase 3B bonds by 0.012. This represents a 97% drop from the average level before dissemination.

In summary, Phase 2 bonds are the most actively traded bonds in the overall inter-dealer trade, the principal inter-dealer trade, and the agency inter-dealer trade. DID estimates show that TRACE does not have a significant effect on the trading frequency of Phase 2 bonds. Phase 3A bonds are similarly thinly traded as Phase 3B bonds in the agency inter-dealer trade. DID estimates show that TRACE significantly lowers the trading frequency of Phase 2 bonds in agency inter-dealer trade and the magnitude of the negative effect is quantitatively similar to that on Phase 3B bonds. Phase 3B bonds are the most thinly traded bonds in the overall inter-dealer trade, the principal inter-dealer trade, and the agency inter-dealer trade. DID estimates show that TRACE significantly lowers the trading frequency of Phase 3B bonds and the overall negative effect mainly

	Phase 2 num of trades per day	Phase 3A num of trades per day	Phase 3B num of trades per day
Disseminate	-0.0502***	-0.0763***	-0.121**
	(-7.23)	(-8.79)	(-3.11)
Post	0.00458***	0.0118***	0.0148***
	(4.76)	(4.28)	(3.43)
Disseminate×Post	0.000510	-0.0138***	-0.0120**
	(0.37)	(-4.93)	(-2.68)
N	16794	20424	7234

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 9: DID Regression Results for Agency Inter-dealer Trade

comes from the negative effect on the principle inter-dealer trade.

7 Extension

In Section 7.1, I study the relationship between the sequential trading game and the simultaneous trading game. The equivalence between them indicates that the modeling choice of the sequential trading game does not play any role in deciding the outcome in equilibrium, even though the sequential trading game is more natural than the simultaneous trading game.[13]

In Section 7.2, I explicitly model the price-discovery process. The price discovery game shows that the equilibrium prices and quantities of my baseline model can be found via an iterative, decentralized process. It shows that my equilibrium outcome is not specific to the double auction modeling technique, which captures the repeated exchange of limit and market orders (i.e., the offer and acceptance of quotes) within a short time interval across fixed counterparties.

[13] I thank Vincent Glode for making this point to me.

In Section 7.3, I relax the assumption for clients' trading intensity to be fixed. It shows when clients trade for the asset payoff, the information diffusion process is the same as the baseline model, but clients' trading intensity would be lower when the information asymmetry becomes larger. Thus the results in the baseline model still hold and the effect on dealers' trading surplus are amplified via changing clients' trading intensity.

7.1 Payoff Equivalent to the Simultaneous Trading Game

Babus and Kondor (2018) and Babus, Kondor and Wang (2019) use a simultaneous trading model to study the information diffusion in the inter-dealer network. In this paper, I use the sequential trading model mainly for the sake of exposition, and it is more suitable for discussing post-trade transparency policy. Under the assumption that one dealer is informed of private information about the asset payoff and dealers are risk-neutral, the following proposition shows that the sequential trading game is payoff equivalent to the simultaneous trading game.

Proposition 6. *Considering the simultaneous trading game, where dealers' linear demand functions are*

$$\Big\{Q^1_{1,2}(\eta_1, s_1, p_{1,2}), \big(Q^2_{1,2}(\eta_2, p_{1,2}, p_{2,3}), Q^2_{2,3}(\eta_2, p_{1,2}, p_{2,3})\big), ...,$$

$$\big(Q^{n-1}_{n-2,n-1}(\eta_{n-1}, p_{n-2,n-1}, p_{n-1,n}), Q^{n-1}_{n-1,n}(\eta_{n-1}, p_{n-2,n-1}, p_{n-1,n})\big), Q^{n}_{n-1,n}(\eta_n, p_{n-1,n})\Big\},$$

the simultaneous trading game shown in the main text is payoff equivalent to the simultaneous trading game.

For any dealer $i \in \{2, ..., n-1\}$, the trading price on link $(i, i+1)$ is not informative about the asset payoff, as the subsequent dealers do not have private information about the asset payoff. Thus, as dealers are risk neutral, the optimization problem of trading on the link $(i-1, i)$ is separate from that on the link $(i, i+1)$, and is not affected by the price on the link $(i, i+1)$. Thus the solution to the sequential trading game I characterize above is also the solution to the simultaneous trading game.

7.2 The Price Discovery Game

In real-world OTC markets, dealers engage in bilateral negotiations with their counterparties by quoting prices which are valid for a certain quantity. To capture this feature, I introduce the price discovery game, as a variant of the OTC game where dealers find the equilibrium prices and quantities through a sequence of bilateral exchange of quotes. It is in the spirit of the price discovery game in Babus and Kondor (2018).

Formally, I define the price-discovery game as follows. In round 0, dealer i chooses a bidding strategy $\pi^i_{i,i+1,0} = \{p^i_{i,i+1,0}, q^i_{i,i+1,0}\}$, and dealer $i+1$ chooses a bidding strategy $\pi^{i+1}_{i,i+1,0} = \{p^{i+1}_{i,i+1,0}, q^{i+1}_{i,i+1,0}\}$. They also choose $B^i_{i,i+1}(s_i, \pi^{i+1}_{i,i+1,\tau}) = \pi^i_{i,i+1,\tau+1}$, which describes the counter-offers that dealer i makes in round $\tau+1$, conditional on the bids it received in round τ. $B^{i+1}_{i,i+1}(s_i, \pi^i_{i,i+1,\tau}) = \pi^{i+1}_{i,i+1,\tau+1}$ describes the counter-offers that dealer $i+1$ makes in round $\tau+1$, conditional on the bids it received in round τ. If there exists a price and quantity vector $\{\bar{p}^i_{i,i+1}, \bar{q}^{i+1}_{i,i+1}\}$ with

$$\bar{p}^i_{i,i+1} = \bar{p}^{i+1}_{i,i+1},$$

$$\bar{q}^i_{i,i+1} + \bar{q}^{i+1}_{i,i+1} + \beta \bar{p}^i_{i,i+1} = 0,$$

and

$$\lim_{\tau \to \infty} \pi^i_{i,i+1,\tau} = (\bar{p}^i_{i,i+1}, \bar{q}^i_{i,i+1}),$$

for some random starting vector $\{\pi^i_{i,i+1,0}, \pi^{i+1}_{i,i+1,0}\}$, then trade takes place.

The payoff for dealer i is $\mathbb{E}[\bar{q}^i_{i,i+1}(\theta + \eta_i - \bar{p}^i_{i,i+1})|s_i]$, provided $\{\bar{p}^i_{i,i+1}, \bar{q}^{i+1}_{i,i+1}\}$ exists, and minus infinity otherwise. Thus, taking dealer $i+1$'s bidding strategy as given, dealer i solves

$$\max_{B^i_{i,i+1}(s_i, \pi^{i+1}_{i,i+1,\tau})} \mathbb{E}[\bar{q}^i_{i,i+1}(\theta + \eta_i - \bar{p}^i_{i,i+1})|s_i, \eta_i];$$

and taking dealer i's bidding strategy as given, dealer $i+1$ solves

$$\max_{B^{i+1}_{i,i+1}(\pi^i_{i,i+1,\tau})} \mathbb{E}[\bar{q}^{i+1}_{i,i+1}(\theta + \eta_i - \bar{p}^i_{i,i+1})|\eta_{i+1}].$$

The following proposition proves that dealers can find the equilibrium prices and quantities in the OTC game by playing the price-discovery game.

Proposition 7. *There exists an equilibrium in the price-discovery game, where prices and quantities are the same as the equilibrium prices and quantities in the equilibrium of the OTC game.*

7.3 Clients Trade for the Asset Payoff

I extend the baseline model of clients such that clients trade not only because of liquidity needs, but also for the asset payoff.

In the trade between dealer i and $i+1$, a continuum of clients participate. Clients do not have private information about the asset payoff or the dealers' private values of the asset. Clients are risk-neutral, Client j's value of the asset is $\theta + \eta_j$ with $\mathbb{E}(\eta_j) = 0$ and η_j is independently distributed across the clients on the same link. Clients incur a quadratic flow cost of trading the asset.

The demand function of client j is $q^j_{i,i+1}(p_{i,i+1}, \eta_j, S)$. The payoff of client j is

$$\mathbb{E}\big((\theta + \eta_j)q^j_{i,i+1} - p_{i,i+1}q^j_{i,i+1} - \frac{\mu}{2}(q^j_{i,i+1})^2|p_{i,i+1}, S\big).$$

Thus we have that the demand of client j is

$$q^j_{i,i+1}(p_{i,i+1}, \eta_j, S) = \frac{1}{\mu}(\eta_j + \mathbb{E}(\theta|p_{i,i+1}, S) - p_{i,i+1}).$$

Thus the aggregate demand of clients on each link $(i, i+1)$ is $\frac{1}{\mu}(\mathbb{E}(\theta|p_{i,i+1}, S) - p_{i,i+1}) = \beta_{i,i+1}p_{i,i+1} + \delta_{i,i+1}S$.

Proposition 8. *In the Linear Bayesian Nash equilibrium of this model,*

1. the information diffusion process is the same as the Bayesian Nash equilibrium of the baseline model;

2. in the trade between dealer i and $i+1$, clients' trading intensity $\beta_{i,i+1}$ and the trading surplus from the asset allocation between dealer i and $i+1$ become smaller if and only if the information asymmetry between dealer i and $i+1$, $Var(\theta|S) - Var(\theta|\hat{s}_i, S)$ becomes larger.

Thus the qualitative results in the baseline model still hold and are amplified due to the effect of information asymmetry on clients' trading intensity.

8 Conclusion

The harm of TRACE has been argued by many market participants. A recent study by Asquith, Covert and Pathak (2019) finds that after transparency, there is a significant decline in the number of trades for thinly traded bonds. Many of the securities markets that are newly subject to transparency are thinly traded. Their empirical results support the view that not every segment of a security market should be subject to the same degree of mandated transparency.

This paper provides a theory framework to fill the gap in the literature in that regard. I build up an information model that features bilateral trade in double auction, endogenous public signal, and inter-dealer network formation to study the effect of TRACE on the inter-dealer markets. For the sake of tractability, I assume there is one dealer in the market that has private information about the asset payoff. In the trading stage, I study the private information diffusion process, and endogenize the public information contained in the disseminated trading price; I show that in markets with a relatively low degree of information asymmetry, post-trade transparency would increase the degree of information asymmetry, and thus make the adverse selection more severe and reduce the surplus from asset reallocation between dealers. The effect of TRACE on dealers' trading profit affects dealers' network decisions and for markets with a low degree of information asymmetry TRACE hurts the network formation and lowers inter-dealer trading frequency. This model implies that the effects of TRACE on inter-dealer trading frequency are not uniform across different markets with different information asymmetry, supported by the empirical evidence in Section 7.

In recent years, TRACE-like transparency policy has been expanding in many other securities markets, e.g., the Agency-Backed Securities market and Asset-Backed Securities required by FINRA, the swaps market required by Title VII of Dodd-Frank Wall Street Reform, etc. This paper sheds light on the discussion whether besides the corporate bond market, other securities markets should be subject to TRACE-like post-trade transparency.

In future research, this framework can be applied to the study of other types of information sharing policies in the OTC markets (for example, pre-trade transparency for bonds brought by MiFID II/R's regulatory regime in Europe, more information sharing of agents' private trading needs in CDS markets[14], etc.) and cast light on their effects on agents' trading behavior and trading surplus, the information diffusion process, and network structure.

[14]Congress passed Section 13 (f) of the Securities Exchange Act in 1975 in order to increase the public availability of information regarding the security holdings of institutional investors. Bethune et al. (2018) interpret the information of institutional investors' security holdings provided to the market in the 13-F form as information related to the trading needs of investors on CDS, as CDS indexes are a way for institutions to hedge against risk in their portfolios.

Reference

Atkeson, Andrew G., Andrea L. Eisfeldt, and Pierre-Olivier Weill, 2015, Entry and Exit in OTC Derivatives Markets *Econometrica*, Vol. 83, 22,31-2292.

Asquith, Paul, Thom Covert, Parag Pathak, 2019, The Effects of Mandatory Transparency in Financial Market Design: Evidence from the Corporate Bond Market, Working Paper.

Babus, Ana, 2016, The Formation of Financial Networks, *RAND Journal of Economics* 47, 2,39-272.

Babus, Ana, and Péter Kondor, 2018, Trading and Information Diffusion in Over-the-Counter Markets, *Econometrica* 86, 1727-1769.

Babus, Ana, Péter Kondor, and Yilin Wang, 2019, Corrigendum to Trading and Information Diffusion in Over-the-Counter Markets, submitted to *Econometrica.*

Bessembinder, H. and W. Maxwell, 2008, Markets: Transparency and the Corporate Bond Market, *Journal of Economic Perspectives* 22 (2), 217-234.

Bessembinder, H., W. Maxwell, and K. Venkataraman, 2006, Market Transparency, Liquidity Externalities, and Institutional Trading Costs in Corporate Bonds, *Journal of Financial Economics* 82, 251-288.

Bethune, Zachary, Bruno Sultanum, and Nicholas Trachter, 2018, An information-based theory of financial intermediation, Working Paper.

Bloomfield, R. and M. O'Hara, 1999, Market Transparency: Who Wins and Who Loses? *The Review of Financial Studies* 12 (1), 5-35.

Blume, Lawrence E., David Easley, Jon Kleinberg, and Eva Tardos, 2009, Trading Networks with Price-Setting Agents, *Games and Economic Behavior* 67, 36-50.

Boyarchenko, Nina, David O. Lucca and Laura Veldkamp, 2018, Taking Orders and Taking Notes: Dealer Information Sharing in Treasury Auctions, Working Paper.

Brancaccio, G., D. Li, and N. Schurhoff, 2017, Learning by Trading: The Case of the U.S. Market for Municipal Bonds, Working Paper.

Chang, Briana and Shengxing Zhang, 2019, Endogenous Market Making and Network Formation, Working Paper.

Demarzo, Péter and Darrell Duffie, 1999, A Liquidity-Based Model of Security Design, *Econometrica* 67, 65-99.

Dick-Nielsen, Jens, 2014, How to Clean Enhanced TRACE Data.

Di Maggio, Franzoni, Kermani, and Sommavilla, 2017, The Relevance of Broker Networks for Information Diffusion in the Stock Market, Working Paper.

Du, Songzhi and Zhu Haoxiang, 2017,What is the Optimal Trading Frequency in Financial Markets? *Review of Economic Studies* 84, 1606-1651.

Duffie, Darrell, Nicholae Garleanu, and Lasse Heje Pedersen, 2005, Over-the-Counter Markets, *Econometrica* 73, 1815-1847.

Duffie, Darrell, Nicolae Garleanu, and Lasse Heje Pedersen, 2007, Valuation in Over-the-Counter Markets, *Review of Financial Studies* 20, 1865-1900.

Edwards, A., L. Harris, and M. Piwowar, 2007, Corporate Bond Market Transparency and Transaction Costs, *Journal of Finance* 62 (3), 1421-1451.

Elliott, Matthew, Benjamin Golub, and Matthew O. Jackson, 2013, Financial Networks and Contagion, Working Paper.

Fishman, M. J. and F. A. Longstaff, 1992, Dual trading in futures market, *The Journal of Finance* 47 (2), 643-671.

Gale, Douglas M., and Shachar Kariv, 2007, Financial Networks, *American Economic Review* 97, 99-103.

Gofman, Michael, 2011, A Network-Based Analysis of Over-the-Counter Markets, Working Paper.

Goldstein, M., E. Hotchkiss, and E. Sirri, 2007, Transparency and Liquidity: A Controlled Experiment on Corporate Bonds, *Review of Financial Studies* 20 (2), 235-273.

Green, T. Clifton, 2004, Economic News and the Impact of Trading on Bond Prices, *Journal of Finance* 59 (3), 1201-1233.

Green, R., B. Hollifield, and N. Schrhoff, 2007a, Financial Intermediation and the Costs of Trading in an Opaque Market, *Review of Financial Studies* 20 (2), 275-314.

Han, Bing, and Liyan Yang, 2013, Social Networks, Information Acquisition, and Asset Prices, *Management Science* 59, 1444-1457.

Hollifield, Burton, Artem Neklyudov, and Chester Spatt, 2017, Bid-ask spreads, trading networks, and the pricing of securitizations, *Review of Financial Studies* 30, 3048-3085.

Kim, Oliver, and Robert E. Verrecchia, 1994, Market liquidity and volume around earnings announcements, *Journal of Accounting and Economics.* 17, 41-67.

Kim, Oliver, and Robert E. Verrecchia, 1997, Pre-announcement and event-period information, *Journal of Accounting and Economics* 24, 395-419.

Kyle, Albert S., 1989, Informed Speculation with Imperfect Competition, *Review of Economic Studies* 56 (3), 317-355.

Li, Dan and Norman Schurhoff 2019, Dealer Networks, *Journal of Finance* 74, 91-144.

Madhavan, A, 1995, Consolidation, Fragmentation, and the Disclosure of Trading Information, *Review of Financial Studies* 8 (3), 579-603.

Malamud, Semyon and Marzena Rostek, 2017, Decentralized Exchange, *American Economic Review* 107, 3320-3362.

Mullen, D., 2004, Relating to Proposed Amendments to TRACE Rule 6250 and Related TRACE Rules to Disseminate Transaction Information on All TRACE-Eligible Securities and Facilitate Dissemination, Letter from Bond Market Association to SEC, July 23.

Naik, N., Neuberger, A., and S. Viswanathan, 1999, Trade Disclosure Regulation in Markets with Negotiated Trades. *Review of Financial Studies* 12 (4), 873-900.

Pagano, M. and A. Roell, 1996, Transparency and Liquidity: A Comparison of Auction and Dealer Markets with Informed Trading, *Journal of Finance* 51 (2), 579-611.

Roell, A., 1990, Dual-capacity trading and the quality of the market, *Journal of Financial Intermediation*, 1 (2), 105-124.

Rostek, M. and Iretka, M., 2012, Price Inference in Small Markets, *Econometrica* 80, 687-711.

Schultz, P., 2012, The Market for New Issues of Municipal Bonds: The Roles of Transparency and Limited Access to Retail Investors, *Journal of Financial Economics* 106, 492-512.

Vives, Xavier, 2011, Strategic Supply Function Competition With Private Information, *Econometrica* 79, 1919-1966.

Zhong, Zhuo and Kei Kawakami, 2016, The Risk Sharing Benefit versus the Collateral Cost: The Formation of the Inter-Dealer Network in Over-the-Counter Trading, Working Paper.

Appendix

Microfoundation of Clients' Linear Demand

In the trade between dealer i and $i+1$, a continuum of clients participate in this trade. Clients do not have private information about the asset payoff or the dealers' private values of the asset. Clients are risk-neutral and have liquidity needs of the asset. Client j's value of the asset is η_j with $\mathbb{E}(\eta_j) = 0$ and independently distributed across the clients. Clients incur a quadratic flow cost of trading the asset.

The demand function of client j is $q^j_{i,i+1}(p_{i,i+1}, \eta_j)$.[15]

The payoff of client j is

$$\mathbb{E}\big(\eta_j q^j_{i,i+1} - p_{i,i+1} q^j_{i,i+1} - \frac{\mu}{2}(q^j_{i,i+1})^2\big)$$

Thus we have that the demand of client j is

$$q^j_{i,i+1}(p_{i,i+1}, \eta_j) = \frac{1}{\mu}(\eta_j - p_{i,i+1})$$

Thus the aggregate demand of clients on each link $(i, i+1)$ is $\beta p_{i,i+1} \equiv -\frac{2}{\mu} p_{i,i-1}$.

It generates a linear demand function of clients, which ensures the existence of linear equilibrium for bilateral trade in double auction. So this provides a micro-foundation for the reduced-form linear demand function of the customer base in Babus and Kondor (2018), Babus, Kondor and Wang (2019).

By assuming that clients' value of the asset does not depend on θ, I shut down clients' learning of the asset payoff from the trading price. In the extension section, I relax the baseline assumption that clients are liquidity traders and assume that clients trade also for the asset payoff. In that case, the information diffusion in the tree network and the equilibrium price function of each link is the same as the baseline model. The details are shown in the proof of Proposition 8. But as clients learn from the price, clients' trading intensity is affected by the information content in the price. As shown in Proposition 8, the main result in this paper still holds and is amplified in this case.

[15] The clients can submit their demand schedules via dealers i and $i+1$, who do dual-capacity trading by submitting clients' orders and trading on their own accounts. See Fishman and Longstaff (1992), Roell (1990).

Proof of Proposition 1

Proof.

Dealer 1's trading strategy is $Q^1_{1,2} = a^1_{1,2}s_1 + b^1_{1,2}p + c^1_{1,2}\eta_1$, dealer 2's trading strategy is $Q^2_{1,2} = b^2_{1,2}p + c^2_{1,2}\eta_2$. For dealer 1, market clearing implies

$$Q^1_{1,2} + b^2_{1,2}p + c^2_{1,2}\eta_2 + \beta p = 0$$

thus

$$p = -\frac{c^2_{1,2}\eta_2}{b^2_{1,2} + \beta} - \frac{Q^1_{1,2}}{b^2_{1,2} + \beta} := I_1 + \lambda^1_{1,2}Q^1_{1,2}$$

Dealer 1's optimization problem is

$$\max_{Q^1_{1,2}} (\mathbb{E}(\theta^1_{1,2}|s_1, \eta_1) - p)Q^1_{1,2} = (\mathbb{E}(\theta^1_{1,2}|s_1, \eta_1) - I_1 - \lambda^1_{1,2}Q^1_{1,2})Q^1_{1,2}$$

FOC

$$-2\lambda^1_{1,2}Q^1_{1,2} + \mathbb{E}(\theta^1_{1,2}|s_1, \eta_1) - I_1 = -2\lambda^1_{1,2}Q^1_{1,2} + \mathbb{E}(\theta^1_{1,2}|s_1, \eta_1) - p + \lambda^1_{1,2}Q^1_{1,2} = -\lambda^1_{1,2}Q^1_{1,2} + \mathbb{E}(\theta^1_{1,2}|s_1, \eta_1) - p = 0$$

thus

$$Q^1_{1,2} = \frac{\mathbb{E}(\theta^1_{1,2}|s_1, \eta_1) - p}{\lambda^1_{1,2}}$$

$$\begin{pmatrix} \theta \\ s_1 \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_\theta & \sigma^2_\theta \\ \sigma^2_\theta & \sigma^2_\theta + \sigma^2_\varepsilon \end{pmatrix}\right]$$

thus

$$\mathbb{E}(\theta|s_1) = \frac{\sigma^2_\theta}{\sigma^2_\theta + \sigma^2_\varepsilon}s_1$$

$$\mathbb{E}(\theta^1_{1,2}|s_1, \eta_1) = \frac{\sigma^2_\theta}{\sigma^2_\theta + \sigma^2_\varepsilon}s_1 + \eta_1$$

we have

$$Q^1_{1,2} = -(b^2_{1,2} + \beta)(\frac{\sigma^2_\theta}{\sigma^2_\theta + \sigma^2_\varepsilon}s_1 + \eta_1 - p) \tag{1}$$

For dealer 2, market clearing condition implies

$$Q^2_{1,2} + a^1_{1,2}s_1 + b^1_{1,2}p + c^1_{1,2}\eta_1 + \beta p = 0$$

thus

$$p = -\frac{a^1_{1,2}s_1 + c^1_{1,2}\eta_1}{b^1_{1,2} + \beta} - \frac{Q^2_{1,2}}{b^1_{1,2} + \beta} := I_2 + \lambda^2_{1,2}Q^2_{1,2}$$

Dealer 2's optimization problem is

$$\max_{Q^2_{1,2}}(\mathbb{E}(\theta_2|I_2,\eta_2)-p)Q^2_{1,2}=(\mathbb{E}(\theta_2|I_2,\eta_2)-I_2-\lambda^2_{1,2}Q^2_{1,2})Q^2_{1,2}$$

FOC implies

$$-2\lambda^2_{1,2}Q^2_{1,2}+\mathbb{E}(\theta_2|I_2,\eta_2)-I_2=0$$

$$\begin{pmatrix}\theta\\ I_2(-\frac{b^1_{1,2}+\beta}{a^1_{1,2}})\end{pmatrix}\sim\mathcal{N}\left[\begin{pmatrix}0\\0\end{pmatrix},\ \begin{pmatrix}\sigma^2_\theta & \sigma^2_\theta\\ \sigma^2_\theta & \sigma^2_\theta+\sigma^2_\varepsilon+\frac{(c^1_{1,2})^2}{(a^1_{1,2})^2}\sigma^2_\eta\end{pmatrix}\right]$$

thus

$$\mathbb{E}(\theta|I_2)=\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{(c^1_{1,2})^2}{(a^1_{1,2})^2}\sigma^2_\eta}I_2(-\frac{b^1_{1,2}+\beta}{a^1_{1,2}})$$

$$\mathbb{E}(\theta_2|I_2,\eta_2)=\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{(c^1_{1,2})^2}{(a^1_{1,2})^2}\sigma^2_\eta}I_2(-\frac{b^1_{1,2}+\beta}{a^1_{1,2}})+\eta_2$$

thus

$$-2\lambda^2_{1,2}Q^2_{1,2}+\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{(c^1_{1,2})^2}{(a^1_{1,2})^2}\sigma^2_\eta}I_2(-\frac{b^1_{1,2}+\beta}{a^1_{1,2}})+\eta_2-I_2=0$$

thus

$$-2\lambda^2_{1,2}Q^2_{1,2}+(\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{(c^1_{1,2})^2}{(a^1_{1,2})^2}\sigma^2_\eta}(-\frac{b^1_{1,2}+\beta}{a^1_{1,2}})-1)(p-\lambda^2_{1,2}Q^2_{1,2})+\eta_2=0$$

thus

$$\lambda^2_{1,2}Q^2_{1,2}(\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{(c^1_{1,2})^2}{(a^1_{1,2})^2}\sigma^2_\eta}(-\frac{b^1_{1,2}+\beta}{a^1_{1,2}})+1)=(\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{(c^1_{1,2})^2}{(a^1_{1,2})^2}\sigma^2_\eta}(-\frac{b^1_{1,2}+\beta}{a^1_{1,2}})-1)p+\eta_2$$

we have

$$Q^2_{1,2}=-(b^1_{1,2}+\beta)(\frac{\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{(c^1_{1,2})^2}{(a^1_{1,2})^2}\sigma^2_\eta}(-\frac{b^1_{1,2}+\beta}{a^1_{1,2}})-1}{\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{(c^1_{1,2})^2}{(a^1_{1,2})^2}\sigma^2_\eta}(-\frac{b^1_{1,2}+\beta}{a^1_{1,2}})+1}p+\frac{1}{\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{(c^1_{1,2})^2}{(a^1_{1,2})^2}\sigma^2_\eta}(-\frac{b^1_{1,2}+\beta}{a^1_{1,2}})+1}\eta_2) \quad (2)$$

From (1)(2), we have

$$a^1_{1,2}=-(b^2_{1,2}+\beta)\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon}:=-(b^2_{1,2}+\beta)\frac{1}{1+\gamma}$$

$$b^1_{1,2}=b^2_{1,2}+\beta$$

$$c^1_{1,2}=-(b^2_{1,2}+\beta)$$

$$b^2_{1,2} = -(b^1_{1,2}+\beta)\frac{\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{(c^1_{1,2})^2}{(a^1_{1,2})^2}\sigma^2_\eta}(-\frac{b^1_{1,2}+\beta}{a^1_{1,2}})-1}{\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{(c^1_{1,2})^2}{(a^1_{1,2})^2}\sigma^2_\eta}(-\frac{b^1_{1,2}+\beta}{a^1_{1,2}})+1}$$

$$c^2_{1,2} = -(b^1_{1,2}+\beta)\frac{1}{\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{(c^1_{1,2})^2}{(a^1_{1,2})^2}\sigma^2_\eta}(-\frac{b^1_{1,2}+\beta}{a^1_{1,2}})+1}$$

then we have

$$b^2_{1,2}(\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{(c^1_{1,2})^2}{(a^1_{1,2})^2}\sigma^2_\eta}(-\frac{b^1_{1,2}+\beta}{a^1_{1,2}})+1) = -(b^1_{1,2}+\beta)(\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{(c^1_{1,2})^2}{(a^1_{1,2})^2}\sigma^2_\eta}(-\frac{b^1_{1,2}+\beta}{a^1_{1,2}})-1)$$

thus

$$b^2_{1,2}(\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{\sigma^2_\eta}{(\frac{1}{1+\gamma})^2}}(-\frac{b^2_{1,2}+2\beta}{-(b^2_{1,2}+\beta)\frac{1}{1+\gamma}})+1) = -(b^2_{1,2}+2\beta)(\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{\sigma^2_\eta}{(\frac{1}{1+\gamma})^2}}(-\frac{b^2_{1,2}+2\beta}{-(b^2_{1,2}+\beta)\frac{1}{1+\gamma}})-1)$$

Define $\kappa_1 \equiv \frac{\sigma^2_\eta}{Var(\theta)-Var(\theta|s_1)} = \varphi(1+\gamma)$, we have

$$b^2_{1,2}(\frac{1}{1+\kappa_1}\frac{b^2_{1,2}+2\beta}{b^2_{1,2}+\beta}+1) = -(b^2_{1,2}+2\beta)(\frac{1}{1+\kappa_1}\frac{b^2_{1,2}+2\beta}{b^2_{1,2}+\beta}-1) \tag{3}$$

thus

$$b^2_{1,2}(\frac{1}{1+\kappa_1}(b^2_{1,2}+2\beta)+b^2_{1,2}+\beta) = -(b^2_{1,2}+2\beta)(\frac{1}{1+\kappa_1}(b^2_{1,2}+2\beta)-(b^2_{1,2}+\beta))$$

thus

$$\frac{1}{1+\kappa_1}(b^2_{1,2}+2\beta)b^2_{1,2}+(b^2_{1,2}+\beta)b^2_{1,2} = -(b^2_{1,2}+2\beta)^2\frac{1}{1+\kappa_1}+(b^2_{1,2}+2\beta)(b^2_{1,2}+\beta)$$

thus

$$\frac{1}{1+\kappa_1}(b^2_{1,2}+2\beta)2(b^2_{1,2}+\beta) = (b^2_{1,2}+\beta)2\beta$$

we have

$$b^2_{1,2} = \beta(1+\kappa_1)-2\beta = \beta(\kappa_1-1) = \beta(\varphi(1+\gamma)-1)$$

We require $\lambda^1_{1,2} = -\frac{1}{b^2_{1,2}+\beta} > 0$, $\lambda^2_{1,2} = -\frac{1}{b^1_{1,2}+\beta} > 0$. We immediately have $b^1_{1,2}+\beta = b^2_{1,2}+2\beta = \beta(1+\kappa_1) < 0$, $b^2_{1,2}+\beta < 0$.

Then we have

$$a^1_{1,2} = -\kappa_1\beta\frac{1}{1+\gamma} = -\beta\varphi$$

$$b^1_{1,2} = \kappa_1\beta = \beta\varphi(1+\gamma)$$

$$c^1_{1,2} = -\kappa_1\beta = -\beta\varphi(1+\gamma)$$

$$c^2_{1,2} = b^2_{1,2}\frac{1}{\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{(c^1_{1,2})^2}{(a^1_{1,2})^2}\sigma^2_\eta}(-\frac{b^1_{1,2}+\beta}{a^1_{1,2}})-1} = (\beta(1+\kappa_1)-2\beta)\frac{1}{\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{\sigma^2_\eta}{(\frac{1}{1+\gamma})^2}}(-\frac{\beta(1+\kappa_1)}{-(\beta(1+\kappa_1)-\beta)\frac{1}{1+\gamma}})-1}$$

$$= (\beta(1+\kappa_1)-2\beta)\frac{1}{\frac{1}{1+\kappa_1}\frac{1}{1+\gamma}(-\frac{\beta(1+\kappa_1)}{-(\beta(1+\kappa_1)-\beta)\frac{1}{1+\gamma}})-1} = -\kappa_1\beta = -\beta\varphi(1+\gamma)$$

Using $a^1_{1,2}s_1 + b^1_{1,2}p + c^1_{1,2}\eta_1 + b^2_{1,2}p + c^2_{1,2}\eta_2 + \beta p = 0$, we have

$$p = \frac{\varphi}{2\kappa_1}s_1 + \frac{\eta_1+\eta_2}{2}$$

□

Proof of Lemma 1

Proof.

Following the same procedure in the proof of Proposition 1, we have the counterparts of (1)(2) above,

$$Q^i_{i,i+1} = -(b^{i+1}_{i,i+1}+\beta)(\frac{\sigma^2_\theta}{Var(s_i)}s_i + \eta_i - p)$$

$$Q^{i+1}_{i,i+1} = -(b^i_{i,i+1}+\beta)(\frac{\frac{\sigma^2_\theta}{Var(s_i)+\frac{(c^i_{i,i+1})^2}{(a^i_{i,i+1})^2}\sigma^2_\eta}(-\frac{b^i_{i,i+1}+\beta}{a^i_{i,i+1}})-1}{\frac{\sigma^2_\theta}{Var(s_i)+\frac{(c^i_{i,i+1})^2}{(a^i_{i,i+1})^2}\sigma^2_\eta}(-\frac{b^i_{i,i+1}+\beta}{a^i_{i,i+1}})+1}p + \frac{1}{\frac{\sigma^2_\theta}{\sigma^2_\theta+\sigma^2_\varepsilon+\frac{(c^i_{i,i+1})^2}{(a^i_{i,i+1})^2}\sigma^{i+1}_\eta}(-\frac{b^i_{i,i+1}+\beta}{a^i_{i,i+1}})+1}\eta_{i+1})$$

we have

$$b^i_{i,i+1} = b^{i+1}_{i,i+1} + \beta$$

$$b^{i+1}_{i,i+1} = -(b^i_{i,i+1}+\beta)(\frac{\frac{\sigma^2_\theta}{Var(s_i)+\frac{(c^i_{i,i+1})^2}{(a^i_{i,i+1})^2}\sigma^2_\eta}(-\frac{b^i_{i,i+1}+\beta}{a^i_{i,i+1}})-1}{\frac{\sigma^2_\theta}{Var(s_i)+\frac{(c^i_{i,i+1})^2}{(a^i_{i,i+1})^2}\sigma^2_\eta}(-\frac{b^i_{i,i+1}+\beta}{a^i_{i,i+1}})+1})$$

thus

$$b^{i+1}_{i,i+1}(\frac{\sigma^2_\theta}{Var(s_i)+\frac{(c^i_{i,i+1})^2}{(a^i_{i,i+1})^2}\sigma^2_\eta}(-\frac{b^i_{i,i+1}+\beta}{a^i_{i,i+1}})+1) = -(b^{i+1}_{i,i+1}+2\beta)(\frac{\sigma^2_\theta}{Var(s_i)+\frac{(c^i_{i,i+1})^2}{(a^i_{i,i+1})^2}\sigma^2_\eta}(-\frac{b^i_{i,i+1}+\beta}{a^i_{i,i+1}})-1)$$

thus

$$b^{i+1}_{i,i+1}(\frac{\sigma^2_\theta}{Var(s_i)+\frac{(Var(s_i))^2}{(\sigma^2_\theta)^2}\sigma^2_\eta}(-\frac{b^i_{i,i+1}+\beta}{-(b^{i+1}_{i,i+1}+\beta)\frac{\sigma^2_\theta}{Var(s_i)}})+1)$$

$$= -(b^{i+1}_{i,i+1}+2\beta)(\frac{\sigma^2_\theta}{Var(s_i)+\frac{(Var(s_i))^2}{(\sigma^2_\theta)^2}\sigma^2_\eta}(-\frac{b^i_{i,i+1}+\beta}{-(b^{i+1}_{i,i+1}+\beta)\frac{\sigma^2_\theta}{Var(s_i)}})-1)$$

Using

$$\frac{1}{1+\kappa_i} = \frac{1}{1+\frac{\sigma_\eta^2}{Var(\theta)-Var(\theta|s_i)}} = \frac{1}{1+\frac{\sigma_\eta^2}{\frac{\sigma_\theta^4}{Var(s_i)}}}$$

we have the counterpart of (3) above

$$b_{i,i+1}^{i+1}\left(\frac{1}{1+\kappa_i}\left(\frac{b_{i,i+1}^{i+1}+2\beta}{(b_{i,i+1}^{i+1}+\beta)}\right)+1\right) = -(b_{i,i+1}^{i+1}+2\beta)\left(\frac{1}{1+\kappa_i}\left(\frac{b_{i,i+1}^{i+1}+2\beta}{(b_{i,i+1}^{i+1}+\beta)}\right)-1\right)$$

the rest of the proof is the same as the proof of Proposition 1.

□

Proof of Lemma 2

Proof.

For dealer 2, market clearing condition for link (1,2) implies

$$Q_{1,2}^2 + a_{1,2}^1 s_1 + b_{1,2}^1 p_{1,2} + c_{1,2}^1 \eta_1 + \beta p_{1,2} = 0$$

thus

$$p_{1,2} = -\frac{a_{1,2}^1 s_1 + c_{1,2}^1 \eta_1}{b_{1,2}^1+\beta} - \frac{Q_{1,2}^2}{b_{1,2}^1+\beta} := I_2 + \lambda_{1,2}^2 Q_{1,2}^2$$

For dealer 2, $p_{1,2}$ is informationally equivalent to $s_1 + \frac{c_{1,2}^1}{a_{1,2}^1}\eta_1 = s_1 + \frac{\kappa_1}{\varphi}\eta_1 := s_2$. We have

$$Var(\theta|s_2) = \sigma_\theta^2 - \frac{\sigma_\theta^4}{Var(s_1)+(\frac{\kappa_1}{\varphi})^2\sigma_\eta^2} = \sigma_\theta^2 - \frac{\sigma_\theta^4}{\frac{\kappa_1}{\varphi^2}\sigma_\eta^2+(\frac{\kappa_1}{\varphi})^2\sigma_\eta^2}$$

thus

$$\frac{\sigma_\eta^2}{Var(\theta)-Var(\theta|s_2)} = \frac{\sigma_\eta^2}{\frac{\sigma_\theta^4}{\frac{\kappa_1}{\varphi^2}\sigma_\eta^2+(\frac{\kappa_1}{\varphi})^2\sigma_\eta^2}} = \kappa_1+\kappa_1^2$$

$$\kappa_2 \equiv \frac{\sigma_\eta^2}{Var(\theta)-Var(\theta|s_2)} = \kappa_1+\kappa_1^2$$

Using

$$\kappa_2 = \kappa_1+\kappa_1^2$$
$$\frac{c_{2,3}^2}{a_{2,3}^2} = \frac{\kappa_2}{\varphi}$$

we prove by induction. Suppose for any $2 \le j < i$,

$$\kappa_j = \kappa_{j-1}+\kappa_{j-1}^2$$

$$\frac{c_{j,j+1}^j}{a_{j,j+1}^j} = \frac{\kappa_j}{\varphi}$$

thus for dealer i, $p_{i-1,i}$ is informationally equivalent to $s_{i-1}+\frac{c^{i-1}_{i-1,i}}{a^{i-1}_{i-1,i}}\eta_{i-1}=s_{i-1}+\frac{\kappa_{i-1}}{\varphi}\eta_{i-1}:=$ s_i, thus

$$Var(\theta|s_i)=\sigma_\theta^2-\frac{\sigma_\theta^4}{Var(s_i)}=\sigma_\theta^2-\frac{\sigma_\theta^4}{Var(s_{i-1})+(\frac{\kappa_{i-1}}{\varphi})^2\sigma_\eta^2}$$

then we have

$$\kappa_i=\frac{\sigma_\eta^2}{Var(\theta)-Var(\theta|s_i)}=\frac{\sigma_\eta^2}{\frac{\sigma_\theta^4}{Var(s_{i-1})+(\frac{\kappa_{i-1}}{\varphi})^2\sigma_\eta^2}}=(Var(s_{i-1})+(\frac{\kappa_{i-1}}{\varphi})^2\sigma_\eta^2)\frac{\varphi}{\sigma_\theta^2}$$

$$=(Var(s_{i-2})+(\frac{\kappa_{i-2}}{\varphi})^2\sigma_\eta^2+(\frac{\kappa_{i-1}}{\varphi})^2\sigma_\eta^2)\frac{\varphi}{\sigma_\theta^2}=(Var(s_1)+(\frac{\kappa_1}{\varphi})^2\sigma_\eta^2...+(\frac{\kappa_{i-2}}{\varphi})^2\sigma_\eta^2+(\frac{\kappa_{i-1}}{\varphi})^2\sigma_\eta^2)\frac{\varphi}{\sigma_\theta^2}$$

$$=(\frac{\kappa_1}{\varphi^2}\sigma_\eta^2+(\frac{\kappa_1}{\varphi})^2\sigma_\eta^2...+(\frac{\kappa_{i-2}}{\varphi})^2\sigma_\eta^2+(\frac{\kappa_{i-1}}{\varphi})^2\sigma_\eta^2)\frac{\varphi}{\sigma_\theta^2}=\kappa_{i-1}+\kappa_{i-1}^2$$

□

Proof of Lemma 3

Proof.

Dealer's trading profit is

$$\mathbb{E}[(Q^{i+1}_{i,i+1}(\eta_{i+1}-\eta_i)]+\mathbb{E}[\beta p_{i,i+1}(p_{i,i+1}-\theta-\eta_i)]$$

where

$$Q^{i+1}_{i,i+1}=b^{i+1}_{i,i+1}p_{i,i+1}+c^{i+1}_{i,i+1}\eta_{i+1}$$

$$p_{i,i+1}=\frac{\varphi}{2\kappa_i}s_i+\frac{\eta_i+\eta_{i+1}}{2}$$

$$c^i_{i,i+1}=c^{i+1}_{i,i+1}=-\beta\kappa_i$$

We have

$$\mathbb{E}[p_{i,i+1}(\eta_{i+1}-\eta_i)]=0$$

$$\mathbb{E}[p^2_{i,i+1}]=\frac{\varphi^2}{4\kappa_i^2}Var(s_i)+\frac{\sigma_\eta^2}{2}=\frac{\varphi^2}{4\kappa_i^2}\frac{\kappa_i}{\varphi^2}\sigma_\eta^2+\frac{\sigma_\eta^2}{2}=\frac{\sigma_\eta^2}{4\kappa_i}+\frac{\sigma_\eta^2}{2}$$

thus

$$\mathbb{E}[(Q^{i+1}_{i,i+1}(\eta_{i+1}-\eta_i)]=c^{i+1}_{i,i+1}\sigma_\eta^2=-\beta\sigma_\eta^2\kappa_i$$

$$\mathbb{E}[\beta p_{i,i+1}(p_{i,i+1}-\theta-\eta_i)]=\beta(\frac{\sigma_\eta^2}{4\kappa_i}+\frac{\sigma_\eta^2}{2}-\frac{\varphi}{2\kappa_i}\sigma_\theta^2-\frac{\sigma_\eta^2}{2})=-\beta\frac{\sigma_\eta^2}{4\kappa_i}$$

□

Proof of Lemma 4

Proof.

$$p_{i,i+1} = \varphi\frac{1}{2\kappa_i}s_i + \frac{\eta_i + \eta_{i+1}}{2}$$

thus

$$p_{i,i+1} - p_{i-1,i} = \varphi\frac{1}{2\kappa_i}s_i + \frac{\eta_i + \eta_{i+1}}{2} - (\varphi\frac{1}{2\kappa_{i-1}}s_{i-1} + \frac{\eta_{i-1} + \eta_n}{2})$$

$$= \varphi\frac{1}{2\kappa_i}(s_1+\kappa_1\frac{\sigma_\theta^2}{\sigma_\eta^2}\eta_1+\kappa_2\frac{\sigma_\theta^2}{\sigma_\eta^2}\eta_2+...+\kappa_{i-1}\frac{\sigma_\theta^2}{\sigma_\eta^2}\eta_{i-1})-\varphi\frac{1}{2\kappa_{i-1}}(s_1+\kappa_1\frac{\sigma_\theta^2}{\sigma_\eta^2}\eta_1+\kappa_2\frac{\sigma_\theta^2}{\sigma_\eta^2}\eta_2+...+\kappa_{n-2}\frac{\sigma_\theta^2}{\sigma_\eta^2}\eta_{n-2})$$

$$+\frac{\eta_{i+1} - \eta_{i-1}}{2}$$

$$= \varphi\frac{1}{2}(\frac{1}{\kappa_i} - \frac{1}{\kappa_{i-1}})(s_1 + \kappa_1\frac{\sigma_\theta^2}{\sigma_\eta^2}\eta_1 + ...\kappa_{n-2}\frac{\sigma_\theta^2}{\sigma_\eta^2}\eta_{n-2}) + \frac{1}{2}\frac{\kappa_{i-1}}{\kappa_i}\eta_{i-1} + \frac{\eta_{i+1} - \eta_{i-1}}{2}$$

$$= \varphi\frac{1}{2}(-\frac{1}{\kappa_{i-1}+1})s_1 + \frac{1}{2}(-\frac{1}{\kappa_{i-1}+1})(\kappa_1\eta_1 + ...\kappa_{n-2}\eta_{n-2}) + (\frac{1}{2}\frac{1}{\kappa_{i-1}+1} - \frac{1}{2})\eta_{i-1} + \frac{1}{2}\eta_{i+1}$$

thus

$$Var(p_{i,i+1}-p_{i-1,i}) = (\varphi\frac{1}{2})^2\frac{\sigma_\theta^2(1+\gamma)}{(\kappa_{i-1}+1)^2}+(\frac{1}{2}\frac{1}{\kappa_{i-1}+1})^2(\kappa_1^2+..+\kappa_{n-2}^2)\sigma_\eta^2+(\frac{1}{2}\frac{\kappa_{i-1}}{\kappa_{i-1}+1})^2\sigma_\eta^2+\frac{1}{4}\sigma_\eta^2$$

$$= (\varphi\frac{1}{2})^2\frac{\sigma_\theta^2\kappa_1\frac{\sigma_\theta^2}{\sigma_\eta^2}}{(\kappa_{i-1}+1)^2} + (\frac{1}{2}\frac{1}{\kappa_{i-1}+1})^2(\kappa_1^2 + .. + \kappa_{n-2}^2)\sigma_\eta^2 + (\frac{1}{2}\frac{\kappa_{i-1}}{\kappa_{i-1}+1})^2\sigma_\eta^2 + \frac{1}{4}\sigma_\eta^2$$

$$= \frac{\sigma_\eta^2}{4}\frac{1}{(\kappa_{i-1}+1)^2}(\kappa_1 + \kappa_1^2 + .. + \kappa_{n-2}^2) + (\frac{1}{2}\frac{\kappa_{i-1}}{\kappa_{i-1}+1})^2\sigma_\eta^2 + \frac{1}{4}\sigma_\eta^2$$

$$= \frac{\sigma_\eta^2}{4}\frac{1}{(\kappa_{i-1}+1)^2}\kappa_{i-1}+(\frac{1}{2}\frac{\kappa_{i-1}}{\kappa_{i-1}+1})^2\sigma_\eta^2+\frac{1}{4}\sigma_\eta^2 = \frac{\sigma_\eta^2}{4}\frac{1}{(\kappa_{i-1}+1)^2}\kappa_i+\frac{1}{4}\sigma_\eta^2 = \frac{\sigma_\eta^2}{4}\frac{\kappa_{i-1}}{\kappa_{i-1}+1}+\frac{1}{4}\sigma_\eta^2$$

thus given any κ_1, $Var(p_{i,i+1} - p_{i-1,i})$ is increasing in $i \geq 2$.

From $\mathbb{E}(|x|) = \sigma\sqrt{\frac{2}{\pi}}\exp(-\frac{\mu^2}{2\sigma^2}) + \mu\big(1 - 2\Phi(-\frac{\mu}{\sigma})\big)$

$$\frac{d\mathbb{E}(|x|)}{d\sigma} = \sqrt{\frac{2}{\pi}}\exp(-\frac{\mu^2}{2\sigma^2}) > 0$$

we have $\mathbb{E}(|p_{i,i+1} - p_{i-1,i}|)$ is increasing in $i \geq 2$.

□

Proof of Proposition 2

Proof.

Dealer i's trading strategy is $Q^i_{i,i+1} = a^i_{i,i+1}\hat{s}_i + b^i_{i,i+1}p + c^i_{i,i+1}\eta_i +^i_{i,i+1} S$, dealer $i+1$'s trading strategy is $Q^{i+1}_{i,i+1} = b^{i+1}_{i,i+1}p + c^{i+1}_{i,i+1}\eta_{i+1} + d^{i+1}_{i,i+1}S$. For dealer i, market clearing implies

$$Q^i_{i,i+1} + b^{i+1}_{i,i+1}p + c^{i+1}_{i,i+1}\eta_{i+1} + d^{i+1}_{i,i+1}S + \beta p = 0$$

thus

$$p = -\frac{c^{i+1}_{i,i+1}\eta_{i+1} + d^{i+1}_{i,i+1}S}{b^{i+1}_{i,i+1} + \beta} - \frac{Q^i_{i,i+1}}{b^{i+1}_{i,i+1} + \beta} := I_i + \lambda^i_{i,i+1}Q^i_{i,i+1}$$

Dealer i's optimization problem is

$$\max_{Q^i_{i,i+1}} (\mathbb{E}(\theta_i|\hat{s}_i, \eta_i, S) - p)Q^i_{i,i+1} = (\mathbb{E}(\theta_i|s_i, \eta_i, S) - I_i - \lambda^i_{i,i+1}Q^i_{i,i+1})Q^i_{i,i+1}$$

FOC

$$-2\lambda^i_{i,i+1}Q^i_{i,i+1} + \mathbb{E}(\theta_i|s_i, \eta_i, S) - I_i = -2\lambda^i_{i,i+1}Q^i_{i,i+1} + \mathbb{E}(\theta_i|s_i, \eta_i, S) - p + \lambda^i_{i,i+1}Q^i_{i,i+1}$$

$$= -\lambda^i_{i,i+1}Q^i_{i,i+1} + \mathbb{E}(\theta_i|\hat{s}_i, \eta_i, S) - p = 0$$

thus

$$Q^i_{i,i+1} = \frac{\mathbb{E}(\theta_i|s_i, \eta_i, S) - p}{\lambda^i_{i,i+1}}$$

Using projection theorem,

$$\begin{pmatrix} \theta \\ \hat{s}_i \\ S \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_\theta & \sigma^2_\theta & \sigma^2_\theta \\ \sigma^2_\theta & Var(\hat{s}_i) & Cov(\hat{s}_i, S) \\ \sigma^2_\theta & Cov(\hat{s}_i, S) & Var(S) \end{pmatrix}\right]$$

$$\mathbb{E}(\theta_i|\hat{s}_i, \eta_i) = \frac{\sigma^2_\theta(Var(S) - Cov(\hat{s}_i, S))}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}\hat{s}_i + \frac{\sigma^2_\theta(Var(\hat{s}_i) - Cov(\hat{s}_i, S))}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}S + \eta_i$$

we have

$$Q^i_{i,i+1} = -(b^{i+1}_{i,i+1}+\beta)(\frac{\sigma^2_\theta(Var(S) - Cov(\hat{s}_i, S))}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}\hat{s}_i + \frac{\sigma^2_\theta(Var(\hat{s}_i) - Cov(\hat{s}_i, S))}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}S + \eta_i - p)$$

For dealer $i+1$, market clearing condition implies

$$Q^{i+1}_{i,i+1} + a^i_{i,i+1}\hat{s}_i + b^i_{i,i+1}p + c^i_{i,i+1}\eta_i + d^i_{i,i+1}S + \beta p = 0$$

thus

$$p = -\frac{a^i_{i,i+1}\hat{s}_i + c^i_{i,i+1}\eta_i}{b^i_{i,i+1} + \beta} - \frac{d^i_{i,i+1}S}{b^i_{i,i+1} + \beta} - \frac{Q^{i+1}_{i,i+1}}{b^i_{i,i+1} + \beta} := I_{i+1} - \frac{d^i_{i,i+1}S}{b^i_{i,i+1} + \beta} + \lambda^{i+1}_{i,i+1}Q^{i+1}_{i,i+1}$$

Dealer $i+1$'s optimization problem is

$$\max_{Q^{i+1}_{i,i+1}} (\mathbb{E}(\theta_{i+1}|I_{i+1},\eta_{i+1})-p)Q^{i+1}_{i,i+1} = (\mathbb{E}(\theta_{i+1}|I_{i+1},\eta_{i+1})-I_{i+1}+\frac{d^i_{i,i+1}S}{b^i_{i,i+1}+\beta}-\lambda^{i+1}_{i,i+1}Q^{i+1}_{i,i+1})Q^{i+1}_{i,i+1}$$

FOC implies

$$-2\lambda^{i+1}_{i,i+1}Q^{i+1}_{i,i+1}+\mathbb{E}(\theta_{i+1}|I_{i+1},\eta_{i+1})-I_{i+1}+\frac{d^i_{i,i+1}S}{b^i_{i,i+1}+\beta}=0$$

Using projection theorem,

$$\begin{pmatrix} \theta \\ \hat{s}_{i+1}=I_{i+1}(-\frac{b^i_{i,i+1}+\beta}{a^i_{i,i+1}}) \\ S \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix}0\\0\\0\end{pmatrix}, \begin{pmatrix} \sigma^2_\theta & \sigma^2_\theta & \sigma^2_\theta \\ \sigma^2_\theta & Var(\hat{s}_{i+1}) & Cov(\hat{s}_{i+1},S) \\ \sigma^2_\theta & Cov(\hat{s}_{i+1},S) & Var(S)\end{pmatrix}\right]$$

$$\begin{aligned}\mathbb{E}(\theta_{i+1}|\hat{s}_{i+1},\eta_i) &= \frac{\sigma^2_\theta(Var(S)-Cov(\hat{s}_{i+1},S))}{Var(\hat{s}_{i+1})Var(S)-Cov(\hat{s}_{i+1},S)^2}I_{i+1}(-\frac{b^i_{i,i+1}+\beta}{a^i_{i,i+1}})+\frac{\sigma^2_\theta(Var(\hat{s}_{i+1})-Cov(\hat{s}_{i+1},S))}{Var(\hat{s}_{i+1})Var(S)-Cov(\hat{s}_{i+1},S)\\ &= k_1I_{i+1}+k_2S+\eta_{i+1}\end{aligned}$$

thus

$$-2\lambda^{i+1}_{i,i+1}Q^{i+1}_{i,i+1}+k_1I_{i+1}+k_2S+\eta_{i+1}-I_{i+1}+\frac{d^i_{i,i+1}S}{b^i_{i,i+1}+\beta}=0$$

$$-2\lambda^{i+1}_{i,i+1}Q^{i+1}_{i,i+1}+(k_1-1)(p-\lambda^{i+1}_{i,i+1}Q^{i+1}_{i,i+1}+\frac{d^i_{i,i+1}S}{b^i_{i,i+1}+\beta})+k_2S+\frac{d^i_{i,i+1}S}{b^i_{i,i+1}+\beta}+\eta_{i+1}=0$$

$$Q^{i+1}_{i,i+1}(-\lambda^{i+1}_{i,i+1})(1+k_1)+(k_1-1)p+k_2S+k_1\frac{d^i_{i,i+1}S}{b^i_{i,i+1}+\beta}+\eta_{i+1}=0$$

we have

$$Q^{i+1}_{i,i+1}=-(b^i_{i,i+1}+\beta)(\frac{k_1-1}{k_1+1}p+\frac{k_2S+k_1\frac{d^i_{i,i+1}}{b^i_{i,i+1}+\beta}S+\eta_{i+1}}{k_1+1})$$

thus using

$$b^i_{i,i+1}=b^{i+1}_{i,i+1}+\beta$$

$$b^{i+1}_{i,i+1}=-(b^i_{i,i+1}+\beta)\frac{k_1-1}{k_1+1}$$

we have

$$b^i_{i,i+1}=b^{i+1}_{i,i+1}+\beta=\frac{\beta}{k_1}$$

$$c^i_{i,i+1}=c^{i+1}_{i,i+1}=-\frac{\beta}{k_1}$$

we also have

$$k_1=\frac{\sigma^2_\theta(Var(S)-Cov(\hat{s}_{i+1},S)}{Var(\hat{s}_{i+1})Var(S)-Cov(\hat{s}_{i+1},S)^2}(-\frac{b^i_{i,i+1}+\beta}{a^i_{i,i+1}})$$

$$= \frac{\sigma_\theta^2(Var(S) - Cov(\hat{s}_{i+1}, S)}{Var(\hat{s}_{i+1})Var(S) - Cov(\hat{s}_{i+1}, S)^2}\left(-\frac{b_{i,i+1}^{i+1} + 2\beta}{-(b_{i,i+1}^{i+1} + \beta)\frac{\sigma_\theta^2(Var(S) - Cov(\hat{s}_i, S))}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}}\right)$$

$$= \frac{b_{i,i+1}^{i+1} + 2\beta}{b_{i,i+1}^{i+1} + \beta} \frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_{i+1}, S)^2}{Var(\hat{s}_{i+1})Var(S) - Cov(\hat{s}_{i+1}, S)^2}$$

thus

$$\frac{1}{k_1} = \frac{b_{i,i+1}^{i+1} + \beta}{b_{i,i+1}^{i+1} + 2\beta} \frac{Var(\hat{s}_{i+1})Var(S) - Cov(\hat{s}_{i+1}, S)^2}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_{i+1}, S)^2}$$

using

$$Var(\hat{s}_{i+1}) = Var(\hat{s}_i) + \left(\frac{c_{i,i+1}^i}{a_{i,i+1}^i}\right)^2 \sigma_\eta^2$$

$$\frac{c_{i,i+1}^i}{a_{i,i+1}^i} = \frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}{\sigma_\theta^2(Var(S) - Cov(\hat{s}_i, S))}$$

we have

$$\frac{1}{k_1} = \frac{b_{i,i+1}^{i+1} + \beta}{b_{i,i+1}^{i+1} + 2\beta}\left(1 + \left(\frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}{\sigma_\theta^2(Var(S) - Cov(\hat{s}_i, S))}\right)^2 \sigma_\eta^2 \frac{Var(S)}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_{i+1}, S)^2}\right)$$

we have

$$b_{i,i+1}^{i+1} + \beta = \frac{b_{i,i+1}^{i+1} + \beta}{b_{i,i+1}^{i+1} + 2\beta}\left(1 + \left(\frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}{\sigma_\theta^2(Var(S) - Cov(\hat{s}_i, S))}\right)^2 \sigma_\eta^2 \frac{Var(S)}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_{i+1}, S)^2}\right)\beta$$

$$b_{i,i+1}^{i+1} + 2\beta = \beta\left(1 + \frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}{\sigma_\theta^4(Var(S) - Cov(\hat{s}_i, S))^2}\sigma_\eta^2 Var(S)\right)$$

$$b_{i,i+1}^{i+1} + \beta = \beta\frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}{\sigma_\theta^4(Var(S) - Cov(\hat{s}_i, S))^2}\sigma_\eta^2 Var(S)$$

using

$$Var(\theta|S) = \sigma_\theta^2 - \frac{\sigma_\theta^4}{Var(S)}$$

$$Var(\theta|\hat{s}_i, S) = \sigma_\theta^2 - \sigma_\theta^4 \frac{Var(S) - 2Cov(\hat{s}_i, S) + Var(\hat{s}_i)}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}$$

we have

$$Var(\theta|S) - Var(\theta|\hat{s}_i, S) = \sigma_\theta^4 \frac{(Var(S) - Cov(\hat{s}_i, S))^2}{(Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2)Var(S)}$$

thus

$$b_{i,i+1}^{i+1} + \beta = \frac{1}{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}\sigma_\eta^2 \beta$$

thus

$$k_1 = \frac{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}{\sigma_\eta^2}$$

Finally, we can solve

$$d^i_{i,i+1} = -\frac{\beta}{k_1}\frac{Var(\hat{s}_i) - Cov(\hat{s}_i, S)}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}\sigma^2_\theta = -\beta\frac{\sigma^2_\eta}{\sigma^2_\theta}\frac{Var(\hat{s}_i) - Cov(\hat{s}_i, S)}{(Var(S) - Cov(\hat{s}_i, S))^2}Var(S)$$

$$d^{i+1}_{i,i+1} = -\beta(1+\frac{1}{k_1})(\frac{k_1\frac{d^i_{i,i+1}}{b^i_{i,i+1}+\beta} + k_2}{k_1+1}) = -\beta(1+\frac{1}{k_1})(\frac{k_1\frac{-\frac{\beta}{k_1}\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma^2_\theta}{\beta(\frac{1}{k_1}+1)} + k_2}{k_1+1})$$

$$= -\frac{\beta}{k_1}(k_1\frac{-\frac{\beta}{k_1}\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma^2_\theta}{\beta(\frac{1}{k_1}+1)} + k_2) = -\frac{\beta}{k_1}(\frac{-\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma^2_\theta}{(\frac{1}{k_1}+1)} + k_2)$$

$$= \frac{\beta}{1+k_1}\frac{Var(\hat{s}_i) - Cov(\hat{s}_i, S)}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}\sigma^2_\theta - \frac{k_2}{k_1}\beta$$

$$= \frac{\beta}{1+k_1}\frac{Var(\hat{s}_i) - Cov(\hat{s}_i, S)}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}\sigma^2_\theta - \frac{\beta}{k_1}\frac{\sigma^2_\theta(Var(\hat{s}_{i+1}) - Cov(\hat{s}_{i+1}, S))}{Var(\hat{s}_{i+1})Var(S) - Cov(\hat{s}_{i+1}, S)^2}$$

$$= \frac{\beta}{1+k_1}\frac{Var(\hat{s}_i) - Cov(\hat{s}_i, S)}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}\sigma^2_\theta - \frac{\beta}{k_1}\frac{\sigma^2_\theta(Var(\hat{s}_i) + (\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma^2_\theta(Var(S)-Cov(\hat{s}_i,S))})^2\sigma^2_\eta - Cov(\hat{s}_i, S}{Var(\hat{s}_i)Var(S) + (\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma^2_\theta(Var(S)-Cov(\hat{s}_i,S))})^2\sigma^2_\eta Var(S) - Co}$$

$$= \frac{\beta}{1+k_1}\frac{Var(\hat{s}_i) - Cov(\hat{s}_i, S)}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}\sigma^2_\theta - \frac{\beta}{k_1}\frac{\sigma^2_\theta(Var(\hat{s}_i) + (\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma^2_\theta(Var(S)-Cov(\hat{s}_i,S))})^2\sigma^2_\eta - Cov(\hat{s}_i, S))}{(Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2)(1 + (\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{(\sigma^2_\theta(Var(S)-Cov(\hat{s}_i,S)))^2}}$$

$$= \frac{\beta}{1+k_1}\frac{Var(\hat{s}_i) - Cov(\hat{s}_i, S)}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}\sigma^2_\theta - \frac{\beta}{k_1}\frac{\sigma^2_\theta(Var(\hat{s}_i) + (\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma^2_\theta(Var(S)-Cov(\hat{s}_i,S))})^2\sigma^2_\eta - Cov(\hat{s}_i, S))}{(Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2)(1+\frac{1}{k_1})}$$

$$= \frac{\beta}{1+k_1}\frac{1}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}\sigma^2_\theta\big(Var(\hat{s}_i) - Cov(\hat{s}_i, S) - Var(\hat{s}_i) - (\frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}{\sigma^2_\theta(Var(S) - Cov(\hat{s}_i, S))}$$

$$= \frac{\beta}{1+k_1}\frac{1}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}\sigma^2_\theta\big(-(\frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}{\sigma^2_\theta(Var(S) - Cov(\hat{s}_i, S))})^2\sigma^2_\eta)\big)$$

$$= -\frac{\beta}{1+k_1}\frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}{(Var(S) - Cov(\hat{s}_i, S))^2}\varphi = -\frac{\beta}{1+k_1}\frac{\sigma^4_\theta}{(Var(\theta|S) - Var(\theta|\hat{s}_i, S))Var(S)}\varphi$$

$$= -\frac{\beta}{1+\frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma^2_\eta}}\frac{\sigma^4_\theta}{(Var(\theta|S) - Var(\theta|\hat{s}_i, S))Var(S)}\varphi$$

□

Proof of Lemma 5

Proof.

With public signal

$$\frac{c^i_{i,i+1}}{a^i_{i,i+1}} = \frac{\sigma^2_\theta}{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}(1 - \frac{Cov(\hat{s}_i, S)}{Var(S)})$$

$$Var(\theta|S) = \sigma^2_\theta - \frac{\sigma^4_\theta}{Var(S)}$$

Using projection theorem,

$$\begin{pmatrix} \theta \\ \hat{s}_{i+1} = \hat{s}_i + \frac{c^i_{i,i+1}}{a^i_{i,i+1}}\eta_i \\ S \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}(1-\frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2 & Cov(\hat{s}_i,S) \\ \sigma_\theta^2 & Cov(\hat{s}_i,S) & Var(S) \end{pmatrix}\right.$$

we have

$$Var(\theta|\hat{s}_{i+1},S) = \sigma_\theta^2 - \frac{\begin{pmatrix}\sigma_\theta^2 & \sigma_\theta^2\end{pmatrix}\begin{pmatrix} Var(S) & -Cov(\hat{s}_i,S) \\ -Cov(\hat{s}_i,S) & Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}(1-\frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2 \end{pmatrix}\begin{pmatrix}\sigma_\theta^2 \\ \sigma_\theta^2\end{pmatrix}}{(Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}(1-\frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2)Var(S) - Cov(\hat{s}_i,S)^2}$$

$$= \sigma_\theta^2 - \sigma_\theta^4\frac{Var(S) + Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}(1-\frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2 - 2Cov(\hat{s}_i,S)}{(Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}(1-\frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2)Var(S) - Cov(\hat{s}_i,S)^2}$$

Similarly,

$$\begin{pmatrix} \theta \\ \hat{s}_i \\ S \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & Var(\hat{s}_i) & Cov(\hat{s}_i,S) \\ \sigma_\theta^2 & Cov(\hat{s}_i,S) & Var(S) \end{pmatrix}\right]$$

we have

$$Var(\theta|\hat{s}_i,S) = \sigma_\theta^2 - \sigma_\theta^4\frac{Var(S) + Var(\hat{s}_i) - 2Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i,S)^2}$$

Define

$$\Delta V \equiv Var(\theta|S) - Var(\theta|\hat{s}_i,S) = -\frac{\sigma_\theta^4}{Var(S)} + \sigma_\theta^4\frac{Var(S) + Var(\hat{s}_i) - 2Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i,S)^2}$$

we have

$$\frac{1}{Var(\theta|S) - Var(\theta|\hat{s}_{i+1},S)} = \frac{1}{\sigma_\theta^4}\frac{1}{\frac{Var(S)+Var(\hat{s}_i)+(\frac{\sigma_\theta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}(1-\frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2-2Cov(\hat{s}_i,S)}{(Var(\hat{s}_i)+(\frac{\sigma_\theta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}(1-\frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2)Var(S)-Cov(\hat{s}_i,S)^2} - \frac{1}{Var(S)}}$$

$$= \frac{1}{\sigma_\theta^4}\frac{(Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{\Delta V}(1-\frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2)Var(S) - Cov(\hat{s}_i,S)^2}{Var(S) + Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{\Delta V}(1-\frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2 - 2Cov(\hat{s}_i,S) - \frac{(Var(\hat{s}_i)+(\frac{\sigma_\theta^2}{\Delta V}(1-\frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2)Var(S)-Cov(\hat{s}_i,}{Var(S)}}$$

$$= \frac{1}{\sigma_\theta^4}\frac{(Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{\Delta V}(1-\frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2)Var(S) - Cov(\hat{s}_i,S)^2}{Var(S) + Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{\Delta V}(1-\frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2 - 2Cov(\hat{s}_i,S) - (Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{\Delta V}(1-\frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2) + C}$$

$$= \frac{1}{\sigma_\theta^4}\frac{(Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{\Delta V}(1-\frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2)Var(S) - Cov(\hat{s}_i,S)^2}{Var(S) - 2Cov(\hat{s}_i,S) + \frac{Cov(\hat{s}_i,S)^2}{Var(S)}}$$

we have

$$\frac{\sigma_\eta^2}{Var(\theta|S) - Var(\theta|\hat{s}_{i+1}, S)} - \frac{\sigma_\eta^2}{\Delta V} - (\frac{\sigma_\eta^2}{\Delta V})^2$$

$$= \sigma_\eta^2 \frac{\frac{1}{\sigma_\theta^4}\Big((Var(\hat{s}_i) + (\frac{\sigma_\theta^2}{\Delta V}(1 - \frac{Cov(\hat{s}_i,S)}{Var(S)}))^2\sigma_\eta^2)Var(S) - Cov(\hat{s}_i, S)^2\Big)\Delta V^2 - \sigma_\eta^2(Var(S) - 2Cov(\hat{s}_i, S) + \frac{Cov}{V}}{(Var(S) - 2Cov(\hat{s}_i, S) + \frac{Cov(\hat{s}_i,S)^2}{Var(S)})\Delta V^2}$$

$$= \sigma_\eta^2 \Delta V \frac{(Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2)(-\frac{1}{Var(S)} + \frac{Var(S)+Var(\hat{s}_i)-2Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}) - \frac{(Var(S)-Cov(\hat{s}_i,S))^2}{Var(S)}}{(Var(S) - 2Cov(\hat{s}_i, S) + \frac{Cov(\hat{s}_i,S)^2}{Var(S)})\Delta V^2}$$

$$= \sigma_\eta^2 \Delta V \frac{\frac{Cov(\hat{s}_i,S)^2 - Var(\hat{s}_i)Var(S) - Var(S)^2 - Cov(\hat{s}_i,S)^2 + 2Var(S)Cov(\hat{s}_i,S)}{Var(S)} + Var(S) + Var(\hat{s}_i) - 2Cov(\hat{s}_i, S)}{(Var(S) - 2Cov(\hat{s}_i, S) + \frac{Cov(\hat{s}_i,S)^2}{Var(S)})\Delta V^2}$$

$$= \sigma_\eta^2 \Delta V \frac{-Var(\hat{s}_i) - Var(S) + 2Cov(\hat{s}_i, S) + Var(S) + Var(\hat{s}_i) - 2Cov(\hat{s}_i, S)}{(Var(S) - 2Cov(\hat{s}_i, S) + \frac{Cov(\hat{s}_i,S)^2}{Var(S)})\Delta V^2} = 0$$

□

Proof of Lemma 6

Proof.

Using the result

$$b_{i,i+1}^i = b_{i,i+1}^{i+1} + \beta = \beta \frac{\sigma_\eta^2}{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}$$

$$c_{i,i+1}^i = c_{i,i+1}^{i+1} = -\beta \frac{\sigma_\eta^2}{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}$$

$$\frac{c_{i,i+1}^i}{a_{i,i+1}^i} = \frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}{\sigma_\theta^2(Var(S) - Cov(\hat{s}_i, S))}$$

$$d_{i,i+1}^i = -\beta \frac{\sigma_\eta^2}{\sigma_\theta^2} \frac{Var(\hat{s}_i) - Cov(\hat{s}_i, S)}{(Var(S) - Cov(\hat{s}_i, S))^2} Var(S)$$

$$d_{i,i+1}^{i+1} = -\frac{\beta}{1 + \frac{Var(\theta|S) - Var(\theta|\hat{s}_i,S)}{\sigma_\eta^2}} \frac{\sigma_\theta^4}{(Var(\theta|S) - Var(\theta|\hat{s}_i, S))Var(S)} \varphi$$

we have

$$p = -\frac{a_{i,i+1}^i}{b_{i,i+1}^i + b_{i,i+1}^{i+1} + \beta}\hat{s}_i - \frac{c_{i,i+1}^i \eta_i + c_{i,i+1}^{i+1}\eta_{i+1}}{b_{i,i+1}^i + b_{i,i+1}^{i+1} + \beta} - \frac{d_{i,i+1}^i + d_{i,i+1}^{i+1}}{b_{i,i+1}^i + b_{i,i+1}^{i+1} + \beta} S$$

$$= \frac{Var(\theta|S) - Var(\theta|\hat{s}_i, S)}{2\sigma_\theta^2(1 - \frac{Cov(\hat{s}_i,S)}{Var(S)})}\hat{s}_i - \frac{d_1 + d_2}{2\sigma_\eta^2 \beta}(Var(\theta|S) - Var(\theta|\hat{s}_i, S))S + \frac{\eta_1 + \eta_2}{2}$$

thus

$$\mathbb{E}[Q_{i,i+1}^{i+1}(\eta_{i+1} - \eta_i)] = \mathbb{E}[(a_i\hat{s}_i + b_{i,i+1}^i p_{i,i+1} + c_{i,i+1}^i \eta_i + d_{i,i+1}^i S)(\eta_{i+1} - \eta_i)] = c_{i,i+1}^i \sigma_\eta^2 = -\beta \frac{\sigma_\eta^4}{Var(\theta|S) - Var(\theta|}$$

$$\mathbb{E}[\beta p_{i,i+1}(p_{i,i+1}-\theta-\eta_i)] = \beta\Big(\frac{(Var(\theta|S)-Var(\theta|\hat{s}_i,S))^2 Var(\hat{s}_i)}{4\sigma_\theta^4(1-\frac{Cov(\hat{s}_i,S)}{Var(S)})^2} - \frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{2(1-\frac{Cov(\hat{s}_i,S)}{Var(S)})}$$

$$+\frac{(d_{i,i+1}^i+d_{i,i+1}^{i+1})^2}{4\sigma_\eta^4\beta^2}(Var(\theta|S)-Var(\theta|\hat{s}_i,S))^2 Var(S) + \frac{d_{i,i+1}^i+d_{i,i+1}^{i+1}}{2\varphi\beta}(Var(\theta|S)-Var(\theta|\hat{s}_i,S))$$

$$-\frac{d_{i,i+1}^i+d_{i,i+1}^{i+1}}{2\sigma_\eta^2\sigma_\theta^2\beta(1-\frac{Cov(\hat{s}_i,S)}{Var(S)})}(Var(\theta|S)-Var(\theta|\hat{s}_i,S))^2 Cov(\hat{s}_i,S)\Big)$$

□

Proof of Lemma 7

Proof.

Dealer n_p learns a private signal $s_{n_p} = \theta + \varepsilon_{n_p}$, where $\gamma_{n_p} \equiv \frac{\sigma_{n_p}^2}{\sigma_\theta^2}$.

Dealer n_p and $n_p + 1$ observe a public signal $S = \theta + \varepsilon_S$, where $\varepsilon_S \sim \mathcal{N}(0, \sigma_p^2)$, $\varepsilon_{n_p} \perp \varepsilon_S$, $\gamma_p \equiv \frac{\sigma_p^2}{\sigma_\theta^2}$.

Without public signal,

$$Var(\theta|s_{n_p}) = \sigma_\theta^2 - \frac{\sigma_\theta^2}{1+\gamma_{n_p}}$$

thus

$$Var(\theta) - Var(\theta|s_1) = \frac{\sigma_\theta^2}{1+\gamma_{n_p}}$$

With public signal,

$$\begin{pmatrix}\theta\\ s_{n_p}\\ S\end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix}0\\0\\0\end{pmatrix}, \begin{pmatrix}\sigma_\theta^2 & \sigma_\theta^2 & \sigma_\theta^2\\ \sigma_\theta^2 & \sigma_\theta^2(1+\gamma_{n_p}) & \sigma_\theta^2\\ \sigma_\theta^2 & \sigma_\theta^2 & \sigma_\theta^2(1+\gamma_p)\end{pmatrix}\right]$$

thus

$$Var(\theta|s_{n_p}, S) = \sigma_\theta^2 - \sigma_\theta^2\frac{\gamma_{n_p}+\gamma_p}{(1+\gamma_{n_p})(1+\gamma_p)-1}$$

$$Var(\theta|S) = \sigma_\theta^2 - \frac{\sigma_\theta^2}{1+\gamma_p}$$

we have

$$Var(\theta|S) - Var(\theta|s_{n_p}, S) = -\frac{\sigma_\theta^2}{1+\gamma_p} + \sigma_\theta^2\frac{\gamma_{n_p}+\gamma_p}{(1+\gamma_{n_p})(1+\gamma_p)-1}$$

thus the change of information asymmetry is

$$Var(\theta|S)-Var(\theta|s_{n_p},S)-(Var(\theta)-Var(\theta|s_{n_p})) = \frac{-2\gamma_{n_p}\gamma_p-\gamma_{n_p}-\gamma_p}{(1+\gamma_{n_p})(1+\gamma_p)((1+\gamma_{n_p})(1+\gamma_p)-1)}\sigma_\theta^2 < 0$$

□

Proof of Lemma 8

Proof.

Using the results in Lemma 6, dealers' profit from asset reallocation is

$$\mathbb{E}(Q^{i+1}_{i,i+1}(\eta_{i+1}-\eta_i)) = \frac{-\beta\sigma_\eta^4}{Var(\theta|S) - Var(\theta|s_{n_p}, S)}$$

thus the increase of profit from asset reallocation is

$$-\beta\sigma_\eta^4 \frac{1}{Var(\theta|S) - Var(\theta|s_{n_p}, S)} + \beta\sigma_\eta^4 \frac{1}{Var(\theta) - Var(\theta|s_{n_p})} > 0$$

we have $\lim_{\gamma_p \to 0} \frac{1}{Var(\theta|S) - Var(\theta|s_{n_p}, S)} = \infty$

From the results in Proposition 2, we have

$$\frac{c^i_{i,i+1}}{b^i_{i,i+1} + b^{i+1}_{i,i+1} + \beta} = \frac{c^{i+1}_{i,i+1}}{b^i_{i,i+1} + b^{i+1}_{i,i+1} + \beta} = -\frac{1}{2}$$

$$\frac{a^i_{i,i+1}}{b^i_{i,i+1} + b^{i+1}_{i,i+1} + \beta} = -\frac{1}{2}\frac{\gamma_p}{(1+\gamma)(1+\gamma_p) - 1}$$

$$\frac{d^i_{i,i+1}}{b^i_{i,i+1} + b^{i+1}_{i,i+1} + \beta} = -\frac{1}{2}\frac{\gamma}{(1+\gamma)(1+\gamma_p) - 1}$$

$$\frac{d^{i+1}_{i,i+1}}{b^i_{i,i+1} + b^{i+1}_{i,i+1} + \beta} = -\frac{1}{2}\frac{((1+\gamma_{n_p})(1+\gamma_p) - 1)^2\varphi}{((1+\gamma_{n_p})\gamma_p^2 + ((1+\gamma_{n_p})(1+\gamma_p) - 1)^2\varphi)(1+\gamma_p) - \gamma_p^2}$$

we have

$$\lim_{\gamma_p \to 0} \frac{a^i_{i,i+1}}{b^i_{i,i+1} + b^{i+1}_{i,i+1} + \beta} = 0$$

$$\lim_{\gamma_p \to 0} \frac{d^i_{i,i+1}}{b^i_{i,i+1} + b^{i+1}_{i,i+1} + \beta} = \lim_{\gamma_p \to 0} \frac{d^{i+1}_{i,i+1}}{b^i_{i,i+1} + b^{i+1}_{i,i+1} + \beta} = -\frac{1}{2}$$

thus

$$\lim_{\gamma_p \to 0} p = \frac{\eta_i + \eta_{i+1}}{2} + \frac{S}{2}$$

thus

$$\lim_{\gamma_p \to 0} \beta\mathbb{E}(p^2 - p\theta - p\eta_i) = \beta(\frac{\sigma_\eta^2}{2} + \frac{\sigma_\theta^2}{4} - \frac{\sigma_\theta^2}{2} - \frac{\sigma_\eta^2}{2}) = -\beta\frac{\sigma_\theta^2}{4}$$

thus we have if γ_p is relatively small, dealers' profit $\mathbb{E}(Q^{i+1}_{i,i+1}(\eta_{i+1}-\eta_i)) + \beta\mathbb{E}(p^2 - p\theta - p\eta_i)$ is larger in the case with public signal.

□

Proof of Proposition 3

Proof.

(1) $n_p = 3$

In the model with transparency, what is public is

$$S \equiv 2(1+\tau)p_{1,2} = s_1 + (1+\tau)(\eta_1 + \eta_2) = s_1 + \frac{\kappa_1}{\varphi}(\eta_1 + \eta_2)$$

from trading with dealer 2, dealer 3 observes $s_3 = s_1 + \frac{\kappa_1}{\varphi}\eta_1 + \frac{\kappa_2}{\varphi}\eta_2$

Without public signal,

$$Var(\theta|s_3) = \sigma_\theta^2 - \frac{\sigma_\theta^2}{\kappa_3}\varphi$$

thus

$$Var(\theta) - Var(\theta|s_3) = \frac{\sigma_\theta^2}{\kappa_3}\varphi$$

With public signal, it's equivalent to dealer 3 observes $s_1 + \frac{\kappa_1}{\varphi}\eta_1$

$$\begin{pmatrix} \theta \\ s_1 + \frac{\kappa_1}{\varphi}\eta_1 \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \frac{\sigma_\eta^2}{\varphi^2}\kappa_2 \end{pmatrix}\right]$$

$$\begin{pmatrix} \theta \\ S \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & \frac{\sigma_\eta^2}{\varphi^2}\kappa_1 + 2\frac{\sigma_\eta^2}{\varphi^2}\kappa_1^2 \end{pmatrix}\right]$$

thus

$$Var(\theta|s_1 + \frac{\kappa_1}{\varphi}\eta_1) = \sigma_\theta^2 - \frac{\sigma_\theta^2}{\kappa_2}\varphi$$

$$Var(\theta|S) = \sigma_\theta^2 - \frac{\sigma_\theta^2}{\kappa_1 + 2\kappa_1^2}\varphi$$

we have

$$Var(\theta|S) - Var(\theta|s_1 + \frac{\kappa_1}{\varphi}\eta_1) = -\frac{\sigma_\theta^2}{\kappa_1 + 2\kappa_1^2}\varphi + \frac{\sigma_\theta^2}{\kappa_2}\varphi$$

thus the change of information asymmetry is

$$Var(\theta|S) - Var(\theta|s_1 + \frac{\kappa_1}{\varphi}\eta_1) - (Var(\theta) - Var(\theta|s_3)) = -\frac{\sigma_\theta^2}{\kappa_1 + 2\kappa_1^2}\varphi + \frac{\sigma_\theta^2}{\kappa_2}\varphi - \frac{\sigma_\theta^2}{\kappa_3}\varphi$$

$$= \sigma_\theta^2\varphi(\frac{1}{\kappa_2} - \frac{1}{\kappa_3} - \frac{1}{\kappa_2 + \kappa_1^2}) = \sigma_\theta^2\varphi\frac{\kappa_2(\kappa_1^2 - 1)}{(\kappa_2 + \kappa_2^2)(\kappa_2 + \kappa_1^2)}$$

we have

$$\mathbb{E}(Q_{3,4}^4(\eta_4 - \eta_3)) = -\beta\sigma_\eta^2(\frac{\kappa_2^2}{\kappa_1^2} + \kappa_2) = -\beta\sigma_\eta^2\varphi\sigma_\theta^2\frac{1}{Var(\theta|S) - Var(\theta|s_1 + \frac{\kappa_1}{\varphi}\eta_1)}$$

thus the increase of profit from asset reallocation is

$$-\beta\sigma_\eta^4\frac{1}{Var(\theta|S)-Var(\theta|s_1+\frac{\kappa_1}{\varphi}\eta_1)}+\beta\sigma_\eta^4\frac{1}{Var(\theta)-Var(\theta|s_3)}>0 \ \text{ iff } \ \kappa_1<1$$

(2) $n_p>3$

Without public signal,

$$Var(\theta|s_n)=\sigma_\theta^2-\frac{\sigma_\theta^2}{\kappa_n}\varphi$$

thus

$$Var(\theta)-Var(\theta|s_n)=\frac{\sigma_\theta^2}{\kappa_n}\varphi$$

with public signal

$$\begin{pmatrix}\theta\\ s_n\\ S\end{pmatrix}\sim\mathcal{N}\left[\begin{pmatrix}0\\0\\0\end{pmatrix},\begin{pmatrix}\sigma_\theta^2 & \sigma_\theta^2 & \sigma_\theta^2\\ \sigma_\theta^2 & \frac{\sigma_\eta^2}{\varphi^2}\kappa_n & \frac{\sigma_\eta^2}{\varphi^2}(\kappa_2+\kappa_1\kappa_2)\\ \sigma_\theta^2 & \frac{\sigma_\eta^2}{\varphi^2}(\kappa_2+\kappa_1\kappa_2) & \frac{\sigma_\eta^2}{\varphi^2}(\kappa_2+\kappa_1^2)\end{pmatrix}\right]$$

thus

$$Var(\theta|s_n,S)=\sigma_\theta^2-\varphi\frac{(\kappa_1^2-\kappa_1\kappa_2)\sigma_\theta^2+(\kappa_n-\kappa_2(1+\kappa_1))\sigma_\theta^2}{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}$$

we have

$$Var(\theta|S)-Var(\theta|s_n,S)=-\frac{\sigma_\theta^2}{\kappa_1+2\kappa_1^2}\varphi+\varphi\frac{(\kappa_1^2-\kappa_1\kappa_2)\sigma_\theta^2+(\kappa_n-\kappa_2(1+\kappa_1))\sigma_\theta^2}{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}$$

thus the change of information asymmetry is

$$Var(\theta|S)-Var(\theta|s_n,S)-(Var(\theta)-Var(\theta|s_n))=$$
$$-\frac{\sigma_\theta^2}{\kappa_1+2\kappa_1^2}\varphi+\varphi\frac{(\kappa_1^2-\kappa_1\kappa_2)\sigma_\theta^2+(\kappa_n-\kappa_2(1+\kappa_1))\sigma_\theta^2}{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}-\frac{\sigma_\theta^2}{\kappa_n}\varphi$$

define

$$M\equiv\frac{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}{(\kappa_n+(\frac{c_n}{a_n})^2\varphi^2)(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}$$

where $\frac{c^n_{n,n+1}}{a^n_{n,n+1}}=\frac{1}{\varphi}\frac{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}{\kappa_1^2-\kappa_1\kappa_2}$.

we have

$$\mathbb{E}(Q^{n+1}_{n,n+1}(\eta_{n+1}-\eta_n))=c_{n+1}\sigma_\eta^2=\beta(1-\frac{1}{M})\sigma_\eta^2=\beta(1-\frac{(\kappa_n+(\frac{c_n}{a_n})^2\varphi^2)(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2})\sigma_\eta^2$$

$$=\beta(1-\frac{(\kappa_n+(\frac{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}{\kappa_1^2-\kappa_1\kappa_2})^2)(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2})\sigma_\eta^2$$

$$=-\beta\frac{(\frac{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}{\kappa_1^2-\kappa_1\kappa_2})^2(\kappa_2+\kappa_1^2)}{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}\sigma_\eta^2=-\beta\frac{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}{(\kappa_1^2-\kappa_1\kappa_2)^2}(\kappa_2+\kappa_1^2)\sigma_\eta^2$$

as

$$Var(\theta|S) - Var(\theta|s_n, S) = \sigma_\theta^2 \varphi \frac{(\kappa_1^2 - \kappa_1\kappa_2)^2}{(\kappa_1 + 2\kappa_1^2)(\kappa_n(\kappa_2 + \kappa_1^2) - (\kappa_2 + \kappa_1\kappa_2)^2)}$$

thus

$$\mathbb{E}(Q_{n,n+1}^{n+1}(\eta_{n+1} - \eta_n)) = -\beta\sigma_\eta^4 \frac{1}{Var(\theta|S) - Var(\theta|s_n, S)}$$

as

$$\frac{1}{Var(\theta|S) - Var(\theta|s_n, S)} - \frac{1}{Var(\theta) - Var(\theta|s_n)} = \sigma_\theta^2\varphi(\frac{\kappa_n(\kappa_2 + \kappa_1^2) - (\kappa_2 + \kappa_1\kappa_2)^2}{(\kappa_1^2 - \kappa_1\kappa_2)^2}(\kappa_2 + \kappa_1^2) - \kappa_n)$$

$$= \sigma_\theta^2\varphi \frac{\kappa_n((\kappa_2 + \kappa_1^2)^2 - (\kappa_1^2 - \kappa_1\kappa_2)^2) - (\kappa_2 + \kappa_1\kappa_2)^2(\kappa_2 + \kappa_1^2)}{(\kappa_1^2 - \kappa_1\kappa_2)^2}$$

$$= \sigma_\theta^2\varphi \frac{\kappa_n(\kappa_2^2 + \kappa_1^4 + 2\kappa_1^2\kappa_2 - (\kappa_1^4 + \kappa_1^2\kappa_2^2 - 2\kappa_1^3\kappa_2)) - (\kappa_1 + \kappa_1\kappa_2)^2(\kappa_2 + \kappa_1^2)}{(\kappa_1^2 - \kappa_1\kappa_2)^2}$$

$$= \sigma_\theta^2\varphi \frac{\kappa_n((\kappa_1 + \kappa_1^2)^2 + 2\kappa_1^2(\kappa_1 + \kappa_1^2) - \kappa_1^2(\kappa_1 + \kappa_1^2)^2 + 2\kappa_1^3(\kappa_1 + \kappa_1^2)) - (\kappa_1 + \kappa_1\kappa_2)^2(\kappa_2 + \kappa_1^2)}{(\kappa_1^2 - \kappa_1\kappa_2)^2}$$

$$= \sigma_\theta^2\varphi \frac{\kappa_n(\kappa_1 + \kappa_1^2)(\kappa_1 + \kappa_1^2 + 2\kappa_1^2 - \kappa_1^3 - \kappa_1^4 + 2\kappa_1^3) - (\kappa_1 + \kappa_1\kappa_2)^2(\kappa_2 + \kappa_1^2)}{(\kappa_1^2 - \kappa_1\kappa_2)^2}$$

$$= \sigma_\theta^2\varphi \frac{\kappa_n(\kappa_1 + \kappa_1^2)(\kappa_1 + 3\kappa_1^2 + \kappa_1^3 - \kappa_1^4) - (\kappa_1 + \kappa_1\kappa_2)^2(\kappa_2 + \kappa_1^2)}{(\kappa_1^2 - \kappa_1\kappa_2)^2}$$

Obviously, it's negative if κ_1 is large enough.

□

Proof of Proposition 4

Proof.

(1) $n_p = 3$

Dealers' profit from serving the clients

$$\mathbb{E}(\beta p(p - \theta - \eta_3)) = \beta\sigma_\eta^2(-\frac{1}{4\kappa_2} + \frac{1}{4}\frac{\kappa_2^2(\kappa_1^2 + \kappa_2)}{(\frac{\kappa_2^2}{\kappa_1^2} + \kappa_2 + 1)^2\kappa_1^4})$$

we have the magnitude of the increase of dealers' profit from serving the clients

$$-\beta\sigma_\eta^2(\frac{1}{4\kappa_2} - \frac{1}{4}\frac{\kappa_2^2(\kappa_1^2 + \kappa_2)}{(\frac{\kappa_2^2}{\kappa_1^2} + \kappa_2 + 1)^2\kappa_1^4} - \frac{1}{4\kappa_3})$$

is smaller than the increase of profit from asset reallocation between dealers when κ_1 is relatively large.

(2) $n_p > 3$

Dealers' profit from serving the clients

$$\mathbb{E}(\beta p(p-\theta-\eta_n)) = \beta\Big(\frac{1}{4}A^2\sigma_\eta^2\kappa_n + \frac{1}{4}B^2\sigma_\eta^2(\kappa_2+\kappa_1^2) + \frac{1}{2}AB\sigma_\eta^2(\kappa_2+\kappa_1\kappa_2) - \frac{1}{2}A\sigma_\eta^2 - \frac{1}{2}B\sigma_\eta^2\Big)$$

where

$$A = \frac{\kappa_1^2-\kappa_1\kappa_2}{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}$$

$$B = \frac{\kappa_n-\kappa_2(1+\kappa_1)}{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2} + \frac{(\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2)^2}{(\kappa_n+(\frac{\kappa_n(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}{\kappa_1^2-\kappa_1\kappa_2})^2)(\kappa_2+\kappa_1^2)-(\kappa_2+\kappa_1\kappa_2)^2}\frac{1}{(\kappa_1^2-\kappa_1\kappa_2)^2}$$

as κ_1 is relatively large, we have $\kappa_n \gg \kappa_1, \kappa_2$, thus $A \to \frac{\kappa_1^2-\kappa_1\kappa_2}{\kappa_n(\kappa_1^2+\kappa_2)}$, $B \to \frac{2}{\kappa_2+\kappa_1^2}$
we have the magnitude of the increase of dealers' profit from serving the clients

$$\beta\sigma_\eta^2\Big(\frac{1}{4}A^2\kappa_n + \frac{1}{4}B^2(\kappa_2+\kappa_1^2) + \frac{1}{2}AB(\kappa_2+\kappa_1\kappa_2) - \frac{1}{2}A - \frac{1}{2}B + \frac{1}{4\kappa_3}\Big)$$

is smaller than the increase of profit from asset reallocation between dealers when κ_1 is relatively large.

□

Proof of Proposition 5

Proof.
(1) For $i \in \{n_p+1, ..., \bar{n}\}$, in equilibrium $\hat{l}_{i-1,i} = 1$, if and only if $C \leq C^*_{i-1,i}(\kappa_1)$, where

$$C^*_{i-1,i}(\kappa_1) \equiv \max\{C^*(i), ..., C^*(\bar{n})\}.$$

For $j \in \{i, ..., \bar{n}\}$,

$$C^*(j) = \min\{\tilde{C}_{1,2}, ..., \tilde{C}_{j-1,j}\}$$

For $k \in \{2, ..., j\}$,

$$\tilde{C}_{k-1,k} = \frac{\mathbb{E}(\pi^{k-1}_{k-1,k}) + \mathbb{E}(\pi^{k}_{k-1,k}) + \frac{1}{2}\big(\mathbb{E}(\pi^{k}_{k,k+1}) + \mathbb{E}(\pi^{k+1}_{k,k+1})\big) + ...\frac{1}{2^{j-k}}\big(\mathbb{E}(\pi^{j-1}_{j-1,j}) + \mathbb{E}(\pi^{j}_{j-1,j})\big)}{1+\frac{1}{2}+...\frac{1}{2^{j-k}}}.$$

As $j \geq i \geq n_p+1$, from Proposition 4, we have when κ_1 is relatively large, $\mathbb{E}(\pi^{j-1}_{j-1,j}) + \mathbb{E}(\pi^{j}_{j-1,j})$ is strictly smaller in the model with $p_{1,2}$ as public signal. Thus $\tilde{C}_{k-1,k}$ is strictly smaller in the model with $p_{1,2}$ as public signal, for $k \in \{2, ..., j\}$. Thus $C^*(j)$ is strictly smaller in the model with $p_{1,2}$ as public signal, for $j \in \{i, ..., \bar{n}\}$. Thus $C^*_{i-1,i}(\kappa_1)$ is strictly smaller in the model with $p_{1,2}$ as public signal, for $i \in \{n_p+1, ..., \bar{n}\}$.
(2) For $i \in \{2, ..., n_p\}$, for $j \geq i \geq 2$, from Proposition 4, we have when κ_1 is relatively large, $\mathbb{E}(\pi^{j-1}_{j-1,j}) + \mathbb{E}(\pi^{j}_{j-1,j})$ is weakly smaller in the model with $p_{1,2}$ as public signal. Thus

$\tilde{C}_{k-1,k}$ is weakly smaller in the model with $p_{1,2}$ as public signal, for $k \in \{2, ..., j\}$. Thus $C^*(j)$ is weakly smaller in the model with $p_{1,2}$ as public signal, for $j \in \{i, ..., \bar{n}\}$. Thus $C^*_{i-1,i}(\kappa_1)$ is weakly smaller in the model with $p_{1,2}$ as public signal, for $i \in \{2, ..., n_p\}$.

□

Proposition 6

Proof.

Suppose there are n dealers in the network. For dealer $n-1$, market clearing condition for the link $(n-2, n-1)$ implies

$$Q^{n-1}_{n-2,n-1} + a_{n-2}s_{n-2} + b_{n-2}p_{n-2,n-1} + c_{n-2}\eta_{n-2} + \beta p_{n-2,n-1} = 0$$

thus

$$p_{n-2,n-1} = -\frac{a_{n-2}s_{n-2} + c_{n-2}\eta_{n-2}}{b_{n-2} + \beta} - \frac{Q^{n-1}_{n-2,n-1}}{b_{n-2} + \beta} := I^{n-1}_{n-2,n-1} + \lambda_{n-1}Q^{n-1}_{n-2,n-1}$$

market clearing condition for the link $(n-1, n)$ implies

$$Q^{n-1}_{n-1,n} + b_n p_{n-1,n} + c_n\eta_n + \beta p_{n-1,n} = 0$$

thus

$$p_{n-1,n} = -\frac{c_n\eta_n}{b_n + \beta} - \frac{Q^{n-1}_{n-1,n}}{b_n + \beta} := I^{n-1}_{n-1,n} + \lambda_{n-1}Q^{n-1}_{n-1,n}$$

The optimization problem of dealer $n-1$ is

$$\max_{Q^{n-1}_{n-2,n-1}, Q^{n-1}_{n-1,n}} (\mathbb{E}(\theta_{n-1}|I^{n-1}_{n-2,n-1}, \eta_{n-1}) - p_{n-2,n-1})Q^{n-1}_{n-2,n-1} + (\mathbb{E}(\theta_{n-1}|I^{n-1}_{n-1,n}, I^{n-1}_{n-2,n-1}, \eta_{n-1}) - p_{n-1,n})Q^{n-1}_{n-1,n}$$

$$= (\mathbb{E}(\theta_{n-1}|I^{n-1}_{n-2,n-1}, \eta_{n-1}) - I^{n-1}_{n-2,n-1} - \lambda_{n-1}Q^{n-1}_{n-2,n-1})Q^{n-1}_{n-2,n-1}$$

$$+(\mathbb{E}(\theta_{n-1}|I^{n-1}_{n-2,n-1}, \eta_{n-1}) - I^{n-1}_{n-1,n} - \lambda_{n-1}Q^{n-1}_{n-1,n})Q^{n-1}_{n-1,n}$$

the above equality uses that $I^{n-1}_{n-1,n}$ is not informative about θ.

Thus it's immediately clear that $Q^{n-1}_{n-2,n-1}$ does not depend on $I^{n-1}_{n-1,n}$, thus does not depend on $p_{n-1,n}$. Going backward, we can see the demand function of any $i \in \{2, ..., n\}$ is $Q^i_{i-1,i}$ just depends on $p_{i-1,i}$ and η_i. Thus the solution to the linear equilibrium of the simultaneous trading game is equivalent to that of the sequential trading game.

□

Proposition 7

Proof.

Starting from an equilibrium in the OTC game, we construct a bidding strategy for dealer i as follows. For dealer i,

$$q^i_{i,i+1,0} = a^i_{i,i+1}s_i + b^i_{i,i+1}p^i_{i,i+1,0} + c^i_{i,i+1}\eta_i$$

$$p^i_{i,i+1,0} = 0$$

$$q^i_{i,i+1,\tau+1} = a^i_{i,i+1}s_i + b^i_{i,i+1}p^i_{i,i+1,\tau+1} + c^i_{i,i+1}\eta_i$$

$$p^i_{i,i+1,\tau+1} = p^{i+1}_{i,i+1,\tau}$$

For dealer $i+1$,

$$q^{i+1}_{i,i+1,0} = b^{i+1}_{i,i+1}p^{i+1}_{i,i+1,0} + c^{i+1}_{i,i+1}\eta_{i+1}$$

$$p^{i+1}_{i,i+1,0} = 0$$

$$q^{i+1}_{i,i+1,\tau+1} = b^{i+1}_{i,i+1}p^i_{i,i+1,\tau+1} + c^{i+1}_{i,i+1}\eta_{i+1}$$

$$p^{i+1}_{i,i+1,\tau+1} = \frac{\varphi}{2\kappa_i}\frac{q^i_{i,i+1,\tau} - b^i_{i,i+1}p^i_{i,i+1,\tau}}{a^i_{i,i+1}} + \frac{\eta_{i+1}}{2}$$

First, we show that if bidding functions are defined as above, the OTC price-discovery process converges to the equilibrium prices and quantities in the OTC game.

In round 1, given the bids in round 0,

$$q^i_{i,i+1,1} = a^i_{i,i+1}s_i + b^i_{i,i+1}p^i_{i,i+1,1} + c^i_{i,i+1}\eta_i$$

$$p^i_{i,i+1,1} = 0$$

$$q^{i+1}_{i,i+1,1} = b^{i+1}_{i,i+1}p^{i+1}_{i,i+1,1} + c^{i+1}_{i,i+1}\eta_i$$

$$p^{i+1}_{i,i+1,1} = \frac{\varphi}{2\kappa_i}(s_i + \frac{\kappa_i}{\varphi}\eta_i) + \frac{\eta_{i+1}}{2}$$

In round 2, given the bids in round 1,

$$q^i_{i,i+1,2} = a^i_{i,i+1}s_i + b^i_{i,i+1}p^i_{i,i+1,2} + c^i_{i,i+1}\eta_i$$

$$p^i_{i,i+1,2} = \frac{\varphi}{2\kappa_i}(s_i + \frac{\kappa_i}{\varphi}\eta_i) + \frac{\eta_{i+1}}{2}$$

$$q^{i+1}_{i,i+1,2} = b^{i+1}_{i,i+1}p^{i+1}_{i,i+1,2} + c^{i+1}_{i,i+1}\eta_{i+1}$$

$$p^{i+1}_{i,i+1,2} = \frac{\varphi}{2\kappa_i}(s_i + \frac{\kappa_i}{\varphi}\eta_i) + \frac{\eta_{i+1}}{2}$$

the trade takes place.

Then I show that dealer i and $i+1$ would not want to change their bidding strategy unilaterally.

Firstly, I show that for dealer $i+1$, given dealer i's strategy, it does not have incentive to deviate. As given dealer i's bid in last round τ, dealer $i+1$ learns $s_i + \frac{\kappa_i}{\varphi}\eta_i$. It can set price $p^{i+1}_{i,i+1,\tau+1}$ such that the trade takes place in next round with $p^{i+1}_{i,i+1,\tau+1}$, $q^{i}_{i,i+1,\tau+2}$, $q^{i+1}_{i,i+1,\tau+2}$, such that

$$q^{i}_{i,i+1,\tau+2} = a^{i}_{i,i+1}s_i + b^{i}_{i,i+1}p^{i+1}_{i,i+1,\tau+1} + c^{i}_{i,i+1}\eta_i$$

$$q^{i+1}_{i,i+1,\tau+2} + q^{i}_{i,i+1,\tau+2} + \beta p^{i+1}_{i,i+1,\tau+1} = 0$$

thus dealer $i+1$ solves

$$\max_{p^{i+1}_{i,i+1,\tau+1}} \mathbb{E}[q^{i+1}_{i,i+1,\tau+2}(\theta + \eta_{i+1} - p^{i+1}_{i,i+1,\tau+1})|s_i + \frac{\kappa_i}{\varphi}\eta_i, \eta_{i+1}]$$

s.t.

$$q^{i+1}_{i,i+1,\tau+2} + a^{i}_{i,i+1}s_i + b^{i}_{i,i+1}p^{i+1}_{i,i+1,\tau+1} + c^{i}_{i,i+1}\eta_i + \beta p^{i+1}_{i,i+1,\tau+1} = 0$$

by construction,

$$a^{i}_{i,i+1} = -\beta\varphi$$

$$b^{i}_{i,i+1} = \beta\kappa_i$$

$$c^{i}_{i,i+1} = -\beta\kappa_i$$

thus

$$\mathbb{E}[q^{i+1}_{i,i+1,\tau+2}(\theta + \eta_{i+1} - p^{i+1}_{i,i+1,\tau+1})|s_i + \frac{\kappa_i}{\varphi}\eta_i, \eta_{i+1}]$$

$$= \mathbb{E}[-(a^{i}_{i,i+1}s_i + b^{i}_{i,i+1}p^{i+1}_{i,i+1,\tau+1} +^{i}_{i,i+1} \eta_i + \beta p^{i+1}_{i,i+1,\tau+1})(\theta + \eta_{i+1} - p^{i+1}_{i,i+1,\tau+1})|s_i + \frac{\kappa_i}{\varphi}\eta_i, \eta_{i+1}]$$

$$= \mathbb{E}[-(-\beta\varphi(s_i + \frac{\kappa_i}{\varphi}\eta_i) + \beta\kappa_i p^{i+1}_{i,i+1,\tau+1} + \beta p^{i+1}_{i,i+1,\tau+1})(\theta + \eta_{i+1} - p^{i+1}_{i,i+1,\tau+1})|s_i + \frac{\kappa_i}{\varphi}\eta_i, \eta_{i+1}]$$

thus dealer $i+1$ solves

$$\max_{p^{i+1}_{i,i+1,\tau+1}} \mathbb{E}[(p^{i+1}_{i,i+1,\tau+1})^2\beta(1+\kappa_i) + p^{i+1}_{i,i+1,\tau+1}(-\beta\varphi(s_i + \frac{\kappa_i}{\varphi}\eta_i) - \beta(1+\kappa_i)(\theta + \eta_{i+1}))|s_i + \frac{\kappa_i}{\varphi}\eta_i, \eta_{i+1}]$$

we have

$$p^{i+1}_{i,i+1,\tau+1} = -\frac{-\beta\varphi(s_i + \frac{\kappa_i}{\varphi}\eta_i) - \beta(1+\kappa_i)(\mathbb{E}(\theta|s_i + \frac{\kappa_i}{\varphi}\eta_i) + \eta_{i+1})}{2\beta(1+\kappa_i)}$$

$$= \frac{\varphi(s_i + \frac{\kappa_i}{\varphi}\eta_i) + (1+\kappa_i)\frac{s_i + \frac{\kappa_i}{\varphi}\eta_i}{\frac{\kappa_{i+1}}{\varphi}}}{2(1+\kappa_i)} + \frac{\eta_{i+1}}{2} = \varphi\frac{1 + \frac{1+\kappa_i}{\kappa_{i+1}}}{2(1+\kappa_i)}(s_i + \frac{\kappa_i}{\varphi}\eta_i) + \frac{\eta_{i+1}}{2}$$

$$= \varphi \frac{\kappa_i(1+\kappa_i)+1+\kappa_i}{2\kappa_i(1+\kappa_i)^2}(s_i + \frac{\kappa_i}{\varphi}\eta_i) + \frac{\eta_{i+1}}{2} = \frac{\varphi}{2\kappa_i}(s_i + \frac{\kappa_i}{\varphi}\eta_i) + \frac{\eta_{i+1}}{2}$$

$$= \frac{\varphi}{2\kappa_i}\frac{q^i_{i,i+1,\tau} - b^i_{i,i+1}p^i_{i,i+1,\tau}}{a_i} + \frac{\eta_{i+1}}{2}$$

Then I show that for dealer i, given dealer $i+1$'s strategy, it does not have incentive to deviate. Given dealer $i+1$'s strategy, the only way dealer i can affect the price that the trade takes place, $\frac{\varphi}{2\kappa_i}\frac{q^i_{i,i+1,\tau}-b^i_{i,i+1}p^i_{i,i+1,\tau}}{a^i_{i,i+1}} + \frac{\eta_{i+1}}{2}$, is to change $\frac{q^i_{i,i+1,\tau}-b^i_{i,i+1}p^i_{i,i+1,\tau}}{a^i_{i,i+1}} := A$. Thus dealer i solves

$$\max_{q^i_{i,i+1,\tau+1}} \mathbb{E}[q^i_{i,i+1,\tau+1}(\theta + \eta_i - \frac{\varphi}{2\kappa_i}A - \frac{\eta_{i+1}}{2})|s_i, \eta_i]$$

s.t.

$$q^i_{i,i+1,\tau+1} + b^{i+1}_{i,i+1}(\frac{\varphi}{2\kappa_i}A + \frac{\eta_{i+1}}{2}) + c^{i+1}_{i,i+1}\eta_{i+1} + \beta(\frac{\varphi}{2\kappa_i}A + \frac{\eta_{i+1}}{2}) = 0$$

thus dealer i solves

$$\max_A \mathbb{E}[-(b^{i+1}_{i,i+1}(\frac{\varphi}{2\kappa_i}A + \frac{\eta_{i+1}}{2}) + c^{i+1}_{i,i+1}\eta_{i+1} + \beta(\frac{\varphi}{2\kappa_i}A + \frac{\eta_{i+1}}{2}))(\theta + \eta_i - \frac{\varphi}{2\kappa_i}A - \frac{\eta_{i+1}}{2})|s_i, \eta_i]$$

by construction,

$$b^{i+1}_{i,i+1} = \beta(\kappa_i - 1)$$

$$c^{i+1}_{i,i+1} = -\beta\kappa_i$$

thus

$$\mathbb{E}[-(b^{i+1}_{i,i+1}(\frac{\varphi}{2\kappa_i}A + \frac{\eta_{i+1}}{2}) + c^{i+1}_{i,i+1}\eta_{i+1} + \beta(\frac{\varphi}{2\kappa_i}A + \frac{\eta_{i+1}}{2}))(\theta + \eta_i - \frac{\varphi}{2\kappa_i}A - \frac{\eta_{i+1}}{2})|s_i, \eta_i]$$

$$= \mathbb{E}[-(\beta(\kappa_i - 1)(\frac{\varphi}{2\kappa_i}A + \frac{\eta_{i+1}}{2}) - \beta\kappa_i\eta_{i+1} + \beta(\frac{\varphi}{2\kappa_i}A + \frac{\eta_{i+1}}{2}))(\theta + \eta_i - \frac{\varphi}{2\kappa_i}A - \frac{\eta_{i+1}}{2})|s_i, \eta_i]$$

$$= -\mathbb{E}[(\beta\kappa_i(\frac{\varphi}{2\kappa_i}A + \frac{\eta_{i+1}}{2}) - \beta\kappa_i\eta_{i+1})(\theta + \eta_i - \frac{\varphi}{2\kappa_i}A - \frac{\eta_{i+1}}{2})|s_i, \eta_i]$$

$$= -\mathbb{E}[(\beta(\frac{\varphi}{2}A - \frac{\kappa_i}{2}\eta_{i+1})(\theta + \eta_i - \frac{\varphi}{2\kappa_i}A - \frac{\eta_{i+1}}{2})|s_i, \eta_i]$$

$$= -\beta\kappa_i\mathbb{E}[-\frac{\varphi^2}{4\kappa_i^2}A^2 + \frac{\varphi}{2\kappa_i}A(\theta + \eta_i) - \frac{\eta_{i+1}}{2}(\theta + \eta_i - \frac{\eta_{i+1}}{2})|s_i, \eta_i]$$

thus

$$A = \frac{\kappa_i}{\varphi}(\mathbb{E}(\theta|s_i, \eta_i) + \eta_i)$$

thus

$$\frac{q^i_{i,i+1,\tau} - b^i_{i,i+1}p^i_{i,i+1,\tau}}{a^i_{i,i+1}} = \frac{\kappa_i}{\varphi}(\mathbb{E}(\theta|s_i, \eta_i) + \eta_i)$$

we have

$$q^i_{i,i+1,\tau} = a^i_{i,i+1}\frac{\kappa_i}{\varphi}(\mathbb{E}(\theta|s_i, \eta_i) + \eta_i) + b^i_{i,i+1}p^i_{i,i+1,\tau} = a^i_{i,i+1}\frac{\kappa_i}{\varphi}(\frac{\varphi}{\kappa_i}s_i + \eta_i) + b^i_{i,i+1}p^i_{i,i+1,\tau}$$

$$= a^i_{i,i+1}(s_i+\frac{\kappa_i}{\varphi}\eta_i)+b^i_{i,i+1}p^i_{i,i+1,\tau} = a^i_{i,i+1}s_i+b^i_{i,i+1}p^i_{i,i+1,\tau}-\beta\kappa_i\eta_i = a^i_{i,i+1}s_i+b^i_{i,i+1}p^i_{i,i+1,\tau}+c^i_{i,i+1}\eta_i$$

□

Proposition 8

Proof.

Dealer i's trading strategy is $Q^i_{i,i+1} = a^i_{i,i+1}\hat{s}_i + b^i_{i,i+1}p + c^i_{i,i+1}\eta_i + d^i_{i,i+1}S$, dealer $i+1$'s trading strategy is $Q^{i+1}_{i,i+1} = b^{i+1}_{i,i+1}p + c^{i+1}_{i,i+1}\eta_{i+1} + d^{i+1}_{i,i+1}S$. Clients' demand is $\beta_{i,i+1}p+\delta_{i,i+1}S$. For dealer i, market clearing implies

$$Q^i_{i,i+1} + b^{i+1}_{i,i+1}p + c^{i+1}_{i,i+1}\eta_{i+1} + d^{i+1}_{i,i+1}S + \beta_{i,i+1}p + \delta_{i,i+1}S = 0$$

thus

$$p = -\frac{c^{i+1}_{i,i+1}\eta_{i+1} + d^{i+1}_{i,i+1}S + \delta_{i,i+1}S}{b^{i+1}_{i,i+1} + \beta_{i,i+1}} - \frac{Q^i_{i,i+1}}{b^{i+1}_{i,i+1} + \beta_{i,i+1}} := I^i_{i,i+1} + \lambda^i_{i,i+1}Q^i_{i,i+1}$$

Dealer i's optimization problem is

$$\max_{Q^i_{i,i+1}} (\mathbb{E}(\theta_i|\hat{s}_i, \eta_i, S) - p)Q^i_{i,i+1} = (\mathbb{E}(\theta_i|s_i, \eta_i, S) - I_i - \lambda^i_{i,i+1}Q^i_{i,i+1})Q^i_{i,i+1}$$

FOC

$$-2\lambda^i_{i,i+1}Q^i_{i,i+1}+\mathbb{E}(\theta_i|s_i,\eta_i,S)-I_i = -2\lambda^i_{i,i+1}Q^i_{i,i+1}+\mathbb{E}(\theta_i|s_i,\eta_i,S)-p+\lambda^i_{i,i+1}Q^i_{i,i+1} = -\lambda^i_{i,i+1}Q^i_{i,i+1}+\mathbb{E}(\theta_i|\hat{s}_i$$

thus

$$Q^i_{i,i+1} = \frac{\mathbb{E}(\theta_i|s_i,\eta_i,S) - p}{\lambda^i_{i,i+1}}$$

Using projection theorem,

$$\begin{pmatrix}\theta \\ \hat{s}_i \\ S\end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix}0 \\ 0 \\ 0\end{pmatrix}, \begin{pmatrix}\sigma^2_\theta & \sigma^2_\theta & \sigma^2_\theta \\ \sigma^2_\theta & Var(\hat{s}_i) & Cov(\hat{s}_i,S) \\ \sigma^2_\theta & Cov(\hat{s}_i,S) & Var(S)\end{pmatrix}\right]$$

$$\mathbb{E}(\theta_i|\hat{s}_i,\eta_i) = \frac{\sigma^2_\theta(Var(S) - Cov(\hat{s}_i,S))}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i,S)^2}\hat{s}_i + \frac{\sigma^2_\theta(Var(\hat{s}_i) - Cov(\hat{s}_i,S))}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i,S)^2}S + \eta_i$$

we have

$$Q^i_{i,i+1} = -(b^{i+1}_{i,i+1}+\beta_{i,i+1})(\frac{\sigma^2_\theta(Var(S) - Cov(\hat{s}_i,S))}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i,S)^2}\hat{s}_i+\frac{\sigma^2_\theta(Var(\hat{s}_i) - Cov(\hat{s}_i,S))}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i,S)^2}S+\eta_i-p)$$

For dealer $i+1$, market clearing condition implies

$$Q_{i,i+1}^{i+1} + a_{i,i+1}^{i}\hat{s}_i + b_{i,i+1}^{i}p + c_{i,i+1}^{i}\eta_i + d_{i,i+1}^{i}S + \beta_{i,i+1}p + \delta_{i,i+1}S = 0$$

thus

$$p = -\frac{a_{i,i+1}^{i}\hat{s}_i + c_{i,i+1}^{i}\eta_i}{b_{i,i+1}^{i} + \beta_{i,i+1}} - \frac{d_{i,i+1}^{i}S + \delta_{i,i+1}S}{b_{i,i+1}^{i} + \beta_{i,i+1}} - \frac{Q_{i,i+1}^{i+1}}{b_{i,i+1}^{i} + \beta_{i,i+1}} := I_{i+1} - \frac{d_{i,i+1}^{i}S + \delta_{i,i+1}S}{b_{i,i+1}^{i} + \beta_{i,i+1}} + \lambda_{i,i+1}^{i+1}Q_{i,i+1}^{i+1}$$

Dealer $i+1$'s optimization problem is

$$\max_{Q_{i,i+1}^{i+1}} (\mathbb{E}(\theta_{i+1}|I_{i+1},\eta_{i+1}) - p)Q_{i,i+1}^{i+1} = (\mathbb{E}(\theta_{i+1}|I_{i+1},\eta_{i+1}) - I_{i+1} + \frac{d_{i,i+1}^{i}S + \delta_{i,i+1}S}{b_{i,i+1}^{i} + \beta_{i,i+1}} - \lambda_{i,i+1}^{i+1}Q_{i,i+1}^{i+1})Q_{i,i+1}^{i+1}$$

FOC implies

$$-2\lambda_{i,i+1}^{i+1}Q_{i,i+1}^{i+1} + \mathbb{E}(\theta_{i+1}|I_{i+1},\eta_{i+1}) - I_{i+1} + \frac{d_{i,i+1}^{i}S + \delta_{i,i+1}S}{b_{i,i+1}^{i} + \beta_{i,i+1}} = 0$$

Using projection theorem,

$$\begin{pmatrix} \theta \\ \hat{s}_{i+1} = I_{i+1}(-\frac{b_{i,i+1}^{i}+\beta_{i,i+1}}{a_{i,i+1}^{i}}) \\ S \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\theta^2 & \sigma_\theta^2 & \sigma_\theta^2 \\ \sigma_\theta^2 & Var(\hat{s}_{i+1}) & Cov(\hat{s}_{i+1},S) \\ \sigma_\theta^2 & Cov(\hat{s}_{i+1},S) & Var(S) \end{pmatrix}\right]$$

$$\begin{aligned} \mathbb{E}(\theta_{i+1}|\hat{s}_{i+1},\eta_i) &= \frac{\sigma_\theta^2(Var(S) - Cov(\hat{s}_{i+1},S))}{Var(\hat{s}_{i+1})Var(S) - Cov(\hat{s}_{i+1},S)^2} I_{i+1}(-\frac{b_{i,i+1}^{i} + \beta_{i,i+1}}{a_{i,i+1}^{i}}) + \frac{\sigma_\theta^2(Var(\hat{s}_{i+1}) - Cov(\hat{s}_{i+1},S}{Var(\hat{s}_{i+1})Var(S) - Cov(\hat{s}_{i+1}} \\ &= k_1 I_{i+1} + k_2 S + \eta_{i+1} \end{aligned}$$

thus

$$-2\lambda_{i,i+1}^{i+1}Q_{i,i+1}^{i+1} + k_1 I_{i+1} + k_2 S + \eta_{i+1} - I_{i+1} + \frac{d_{i,i+1}^{i}S + \delta_{i,i+1}S}{b_{i,i+1}^{i} + \beta_{i,i+1}} = 0$$

$$-2\lambda_{i,i+1}^{i+1}Q_{i,i+1}^{i+1} + (k_1 - 1)(p - \lambda_{i,i+1}^{i+1}Q_{i,i+1}^{i+1} + \frac{d_{i,i+1}^{i}S + \delta_{i,i+1}S}{b_{i,i+1}^{i} + \beta_{i,i+1}}) + k_2 S + \frac{d_{i,i+1}^{i}S + \delta_{i,i+1}S}{b_{i,i+1}^{i} + \beta_{i,i+1}} + \eta_{i+1} = 0$$

$$Q_{i,i+1}^{i+1}(-\lambda_{i,i+1}^{i+1})(1 + k_1) + (k_1 - 1)p + k_2 S + k_1\frac{d_{i,i+1}^{i}S + \delta_{i,i+1}S}{b_{i,i+1}^{i} + \beta_{i,i+1}} + \eta_{i+1} = 0$$

we have

$$Q_{i,i+1}^{i+1} = -(b_{i,i+1}^{i} + \beta_{i,i+1})(\frac{k_1 - 1}{k_1 + 1}p + \frac{k_2 S + k_1\frac{d_{i,i+1}^{i}S + \delta_{i,i+1}S}{b_{i,i+1}^{i} + \beta_{i,i+1}} + \eta_{i+1}}{k_1 + 1})$$

thus using

$$b_{i,i+1}^{i} = b_{i,i+1}^{i+1} + \beta_{i,i+1}$$

$$b_{i,i+1}^{i+1} = -(b_{i,i+1}^{i} + \beta_{i,i+1})\frac{k_1 - 1}{k_1 + 1}$$

we have

$$b^{i}_{i,i+1} = b^{i+1}_{i,i+1} + \beta_{i,i+1} = \frac{\beta_{i,i+1}}{k_1}$$

$$c^{i}_{i,i+1} = c^{i+1}_{i,i+1} = -\frac{\beta_{i,i+1}}{k_1}$$

we also have

$$k_1 = \frac{\sigma_\theta^2(Var(S) - Cov(\hat{s}_{i+1}, S)}{Var(\hat{s}_{i+1})Var(S) - Cov(\hat{s}_{i+1}, S)^2}\left(-\frac{b^{i}_{i,i+1} + \beta_{i,i+1}}{a^{i}_{i,i+1}}\right)$$

$$= \frac{\sigma_\theta^2(Var(S) - Cov(\hat{s}_{i+1}, S)}{Var(\hat{s}_{i+1})Var(S) - Cov(\hat{s}_{i+1}, S)^2}\left(-\frac{b^{i+1}_{i,i+1} + 2\beta_{i,i+1}}{-(b^{i+1}_{i,i+1} + \beta_{i,i+1})\frac{\sigma_\theta^2(Var(S) - Cov(\hat{s}_i, S))}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}}\right)$$

$$= \frac{b^{i+1}_{i,i+1} + 2\beta_{i,i+1}}{b^{i+1}_{i,i+1} + \beta_{i,i+1}} \frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_{i+1}, S)^2}{Var(\hat{s}_{i+1})Var(S) - Cov(\hat{s}_{i+1}, S)^2}$$

thus

$$\frac{1}{k_1} = \frac{b^{i+1}_{i,i+1} + \beta_{i,i+1}}{b^{i+1}_{i,i+1} + 2\beta_{i,i+1}} \frac{Var(\hat{s}_{i+1})Var(S) - Cov(\hat{s}_{i+1}, S)^2}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_{i+1}, S)^2}$$

using

$$Var(\hat{s}_{i+1}) = Var(\hat{s}_i) + \left(\frac{c^{i}_{i,i+1}}{a^{i}_{i,i+1}}\right)^2\sigma_\eta^2$$

$$\frac{c^{i}_{i,i+1}}{a^{i}_{i,i+1}} = \frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}{\sigma_\theta^2(Var(S) - Cov(\hat{s}_i, S))}$$

we have

$$\frac{1}{k_1} = \frac{b^{i+1}_{i,i+1} + \beta_{i,i+1}}{b^{i+1}_{i,i+1} + 2\beta_{i,i+1}}\Big(1 + \left(\frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}{\sigma_\theta^2(Var(S) - Cov(\hat{s}_i, S))}\right)^2\sigma_\eta^2 \frac{Var(S)}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_{i+1}, S)^2}\Big)$$

we have

$$b^{i+1}_{i,i+1} + \beta_{i,i+1} = \frac{b^{i+1}_{i,i+1} + \beta_{i,i+1}}{b^{i+1}_{i,i+1} + 2\beta_{i,i+1}}\Big(1 + \left(\frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}{\sigma_\theta^2(Var(S) - Cov(\hat{s}_i, S))}\right)^2\sigma_\eta^2 \frac{Var(S)}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_{i+1}, S)^2}\Big)\beta_{i,}$$

$$b^{i+1}_{i,i+1} + 2\beta_{i,i+1} = \beta_{i,i+1}\Big(1 + \frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}{\sigma_\theta^4(Var(S) - Cov(\hat{s}_i, S))^2}\sigma_\eta^2 Var(S)\Big)$$

$$b^{i+1}_{i,i+1} + \beta_{i,i+1} = \beta_{i,i+1}\frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}{\sigma_\theta^4(Var(S) - Cov(\hat{s}_i, S))^2}\sigma_\eta^2 Var(S)$$

using

$$Var(\theta|S) = \sigma_\theta^2 - \frac{\sigma_\theta^4}{Var(S)}$$

$$Var(\theta|\hat{s}_i, S) = \sigma_\theta^2 - \sigma_\theta^4\frac{Var(S) - 2Cov(\hat{s}_i, S) + Var(\hat{s}_i)}{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2}$$

we have

$$Var(\theta|S) - Var(\theta|\hat{s}_i, S) = \sigma_\theta^4\frac{(Var(S) - Cov(\hat{s}_i, S))^2}{(Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i, S)^2)Var(S)}$$

thus

$$b_{i,i+1}^{i+1}+\beta_{i,i+1}=\frac{1}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}\sigma_\eta^2\beta_{i,i+1}$$

thus

$$k_1=\frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma_\eta^2}$$

Finally, we can solve

$$d_{i,i+1}^{i}=-\frac{\beta_{i,i+1}}{k_1}\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2=-\beta_{i,i+1}\frac{\sigma_\eta^2}{\sigma_\theta^2}\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{(Var(S)-Cov(\hat{s}_i,S))^2}Var(S)$$

$$d_{i,i+1}^{i+1}=-\beta_{i,i+1}(1+\frac{1}{k_1})(\frac{k_1\frac{d_{i,i+1}^{i}+\lambda_{i,i+1}^{i}}{b_{i,i+1}^{i}+\beta_{i,i+1}}+k_2}{k_1+1})=-\beta_{i,i+1}(1+\frac{1}{k_1})(\frac{k_1\frac{-\frac{\beta_{i,i+1}}{k_1}\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2+\lambda_{i,i+1}^{i}}{\beta_{i,i+1}(\frac{1}{k_1}+1)}+k_2}{k_1+1})$$

$$=-\frac{\beta_{i,i+1}}{k_1}(k_1\frac{-\frac{\beta_{i,i+1}}{k_1}\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2+\lambda_{i,i+1}^{i}}{\beta_{i,i+1}(\frac{1}{k_1}+1)}+k_2)=-\frac{\beta_{i,i+1}}{k_1}(\frac{-\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2+\frac{k_1}{\beta_{i,i+1}}\lambda_{i,}^{i}}{(\frac{1}{k_1}+1)})$$

$$=\frac{\beta_{i,i+1}}{1+k_1}(\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2-\frac{k_1}{\beta_{i,i+1}}\lambda_{i,i+1}^{i})-\frac{k_2}{k_1}\beta_{i,i+1}$$

$$=\frac{\beta_{i,i+1}}{1+k_1}(\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2-\frac{k_1}{\beta_{i,i+1}}\lambda_{i,i+1}^{i})-\frac{\beta_{i,i+1}}{k_1}\frac{\sigma_\theta^2(Var(\hat{s}_{i+1})-Cov(\hat{s}_{i+1},S))}{Var(\hat{s}_{i+1})Var(S)-Cov(\hat{s}_{i+1},S)^2}$$

$$=\frac{\beta_{i,i+1}}{1+k_1}(\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2-\frac{k_1}{\beta_{i,i+1}}\lambda_{i,i+1}^{i})-\frac{\beta_{i,i+1}}{k_1}\frac{\sigma_\theta^2(Var(\hat{s}_i)+(\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))})}{Var(\hat{s}_i)Var(S)+(\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))})}$$

$$=\frac{\beta_{i,i+1}}{1+k_1}(\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2-\frac{k_1}{\beta_{i,i+1}}\lambda_{i,i+1}^{i})-\frac{\beta_{i,i+1}}{k_1}\frac{\sigma_\theta^2(Var(\hat{s}_i)+(\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))})}{(Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2)(1+(\frac{Var}{(\sigma_\theta^2}))}$$

$$=\frac{\beta_{i,i+1}}{1+k_1}(\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2-\frac{k_1}{\beta_{i,i+1}}\lambda_{i,i+1}^{i})-\frac{\beta_{i,i+1}}{k_1}\frac{\sigma_\theta^2(Var(\hat{s}_i)+(\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))})^2\sigma_\eta^2}{(Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2)(1}$$

$$=\frac{\beta_{i,i+1}}{1+k_1}\frac{1}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2\big(Var(\hat{s}_i)-Cov(\hat{s}_i,S)-Var(\hat{s}_i)-(\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))}\big)$$

$$-\frac{\beta_{i,i+1}}{1+k_1}\frac{k_1}{\beta_{i,i+1}}\lambda_{i,i+1}^{i}$$

$$=\frac{\beta_{i,i+1}}{1+k_1}\frac{1}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}\sigma_\theta^2\big(-(\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))})^2\sigma_\eta^2)\big)-\frac{k_1}{1+k_1}\lambda_{i,i+1}^{i}$$

$$=-\frac{\beta_{i,i+1}}{1+k_1}\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{(Var(S)-Cov(\hat{s}_i,S))^2}\varphi-\frac{k_1}{1+k_1}\lambda_{i,i+1}^{i}$$

$$=-\frac{\beta_{i,i+1}}{1+k_1}\frac{\sigma_\theta^4}{(Var(\theta|S)-Var(\theta|\hat{s}_i,S))Var(S)}\varphi-\frac{k_1}{1+k_1}\lambda_{i,i+1}^{i}$$

$$=-\frac{\beta_{i,i+1}}{1+\frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma_\eta^2}}\frac{\sigma_\theta^4}{(Var(\theta|S)-Var(\theta|\hat{s}_i,S))Var(S)}\varphi-\frac{\frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma_\eta^2}}{1+\frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma_\eta^2}}\lambda_{i,i+1}^{i}$$

For client,

$$\max_q \mathbb{E}\big((\theta - p)q - \frac{\mu}{2}q^2|p,S\big)$$

thus

$$q = \frac{\mathbb{E}(\theta|p,S) - p}{\mu}$$

using the results above,

$$b^i_{i,i+1} = b^{i+1}_{i,i+1} + \beta_{i,i+1} = \frac{\beta_{i,i+1}}{k_1}$$

$$c^i_{i,i+1} = c^{i+1}_{i,i+1} = -\frac{\beta_{i,i+1}}{k_1}$$

$$\frac{c^i_{i,i+1}}{a^i_{i,i+1}} = \frac{Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i,S)^2}{\sigma^2_\theta(Var(S) - Cov(\hat{s}_i,S))}$$

$$k_1 = \frac{Var(\theta|S) - Var(\theta|\hat{s}_i,S)}{\sigma^2_\eta}$$

$$d^i_{i,i+1} = -\beta_{i,i+1}\frac{\sigma^2_\eta}{\sigma^2_\theta}\frac{Var(\hat{s}_i) - Cov(\hat{s}_i,S)}{(Var(S) - Cov(\hat{s}_i,S))^2}Var(S)$$

$$d^{i+1}_{i,i+1} = -\frac{\beta_{i,i+1}}{1 + \frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma^2_\eta}}\frac{\sigma^4_\theta}{(Var(\theta|S) - Var(\theta|\hat{s}_i,S))Var(S)}\varphi - \frac{\frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma^2_\eta}}{1 + \frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma^2_\eta}}\lambda^i_{i,i+1}$$

we have

$$p = -\frac{a^i_{i,i+1}}{b^i_{i,i+1} + b^{i+1}_{i,i+1} + \beta_{i,i+1}}\hat{s}_i - \frac{c^i_{i,i+1}\eta_i + c^{i+1}_{i,i+1}\eta_{i+1}}{b^i_{i,i+1} + b^{i+1}_{i,i+1} + \beta_{i,i+1}} - \frac{d^i_{i,i+1} + d^{i+1}_{i,i+1} + \lambda^i_{i,i+1}}{b^i_{i,i+1} + b^{i+1}_{i,i+1} + \beta_{i,i+1}}S$$

$$= -\frac{c^i_{i,i+1}\frac{\sigma^2_\theta(Var(S)-Cov(\hat{s}_i,S))}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}}{2\frac{\beta_{i,i+1}}{k_1}}\hat{s}_i - \frac{-\frac{\beta_{i,i+1}}{k_1}}{2\frac{\beta_{i,i+1}}{k_1}}(\eta_i + \eta_{i+1})$$

$$-\frac{-\beta_{i,i+1}\frac{\sigma^2_\eta}{\sigma^2_\theta}\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{(Var(S)-Cov(\hat{s}_i,S))^2}Var(S) - \frac{\beta_{i,i+1}}{1+\frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma^2_\eta}}\frac{\sigma^4_\theta}{(Var(\theta|S)-Var(\theta|\hat{s}_i,S))Var(S)}\varphi - \frac{\frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma^2_\eta}}{1+\frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma^2_\eta}}\lambda}{2\frac{\beta_{i,i+1}}{k_1}}$$

$$= \frac{\frac{\sigma^2_\theta(Var(S)-Cov(\hat{s}_i,S))}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}}{2}\hat{s}_i + \frac{1}{2}(\eta_i + \eta_{i+1})$$

$$-\frac{-\beta_{i,i+1}\frac{\sigma^2_\eta}{\sigma^2_\theta}\frac{Var(\hat{s}_i)-Cov(\hat{s}_i,S)}{(Var(S)-Cov(\hat{s}_i,S))^2}Var(S) - \frac{\beta_{i,i+1}}{1+\frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma^2_\eta}}\frac{\sigma^4_\theta}{(Var(\theta|S)-Var(\theta|\hat{s}_i,S))Var(S)}\varphi + \frac{1}{1+\frac{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}{\sigma^2_\eta}}\lambda}{2\frac{\beta_{i,i+1}}{k_1}}$$

$$:= x\hat{s}_i + yS + \frac{\eta_i + \eta_{i+1}}{2}$$

$$\begin{pmatrix} \theta \\ \frac{p-yS}{x} \\ S \end{pmatrix} \sim \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_\theta & \sigma^2_\theta & \sigma^2_\theta \\ \sigma^2_\theta & Var(\hat{s}_i) + \frac{1}{2x^2}\sigma^2_\eta & Cov(\hat{s}_i,S) \\ \sigma^2_\theta & Cov(\hat{s}_i,S) & Var(S) \end{pmatrix}\right]$$

thus

$$\mathbb{E}(\theta|p,S) = \sigma_\theta^2 \frac{(Var(S) - Cov(\hat{s}_i,S))\frac{p-yS}{x} + (Var(\hat{s}_i) + \frac{1}{2x^2}\sigma_\eta^2 - Cov(\hat{s}_i,S))S}{Var(S)(Var(\hat{s}_i) + \frac{1}{2x^2}\sigma_\eta^2) - Cov(\hat{s}_i,S)^2}$$

thus

$$\beta_{i,i+1} = (\sigma_\theta^2 \frac{(Var(S) - Cov(\hat{s}_i,S))\frac{1}{x}}{Var(S)(Var(\hat{s}_i) + \frac{1}{2x^2}\sigma_\eta^2) - Cov(\hat{s}_i,S)^2} - 1)\frac{1}{\mu}$$

$$= (\sigma_\theta^2 \frac{(Var(S) - Cov(\hat{s}_i,S))\frac{1}{\frac{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))}{\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{2}}}}{Var(S)(Var(\hat{s}_i) + \frac{1}{2(\frac{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))}{\frac{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2}{2}})^2}\sigma_\eta^2) - Cov(\hat{s}_i,S)^2} - 1)\frac{1}{\mu}$$

$$= (\frac{2(Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i,S)^2)}{Var(S)(Var(\hat{s}_i) + \frac{2}{(\frac{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2})^2}\sigma_\eta^2) - Cov(\hat{s}_i,S)^2} - 1)\frac{1}{\mu}$$

$$= (\frac{2(Var(\hat{s}_i)Var(S) - Cov(\hat{s}_i,S)^2)}{Var(S)Var(\hat{s}_i) - Cov(\hat{s}_i,S)^2 + Var(S)\frac{2}{(\frac{\sigma_\theta^2(Var(S)-Cov(\hat{s}_i,S))}{Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2})^2}\sigma_\eta^2} - 1)\frac{1}{\mu}$$

$$= (\frac{2}{1 + Var(S)\frac{2(Var(\hat{s}_i)Var(S)-Cov(\hat{s}_i,S)^2)}{\sigma_\theta^4(Var(S)-Cov(\hat{s}_i,S))^2}\sigma_\eta^2} - 1)\frac{1}{\mu}$$

$$= (\frac{2}{1 + 2\frac{\sigma_\eta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}} - 1)\frac{1}{\mu} = (\frac{1 - 2\frac{\sigma_\eta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}}{1 + 2\frac{\sigma_\eta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}})\frac{1}{\mu}$$

$$\lambda_{i,i+1}^i = \sigma_\theta^2 \frac{(Var(S) - Cov(\hat{s}_i,S))\frac{-y}{x} + (Var(\hat{s}_i) + \frac{1}{2x^2}\sigma_\eta^2 - Cov(\hat{s}_i,S))}{Var(S)(Var(\hat{s}_i) + \frac{1}{2x^2}\sigma_\eta^2) - Cov(\hat{s}_i,S)^2}\frac{1}{\mu}$$

thus we have

$$c_{i,i+1}^{i+1} = (\frac{2}{1 + 2\frac{\sigma_\eta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}} - 1)\frac{1}{\mu}(-\frac{\sigma_\eta^2}{Var(\theta|S) - Var(\theta|\hat{s}_i,S)})$$

$$= (1 - \frac{2}{1 + 2\frac{\sigma_\eta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}})\frac{1}{\mu}\frac{\sigma_\eta^2}{Var(\theta|S) - Var(\theta|\hat{s}_i,S)}$$

As the second order condition requires that

$$b_{i,i+1}^{i+1} + \beta_{i,i+1} = \frac{1}{Var(\theta|S) - Var(\theta|\hat{s}_i,S)}\sigma_\eta^2\beta_{i,i+1}$$

$$= \frac{1}{Var(\theta|S) - Var(\theta|\hat{s}_i,S)}\sigma_\eta^2(\frac{1 - 2\frac{\sigma_\eta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}}{1 + 2\frac{\sigma_\eta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}})\frac{1}{\mu} < 0$$

which requires $\frac{\sigma_\eta^2}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)} > \frac{1}{2}$. It's immediately clear that $\beta_{i,i+1}$ is decreasing in $\frac{1}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}$ and the surplus from the asset reallocation $c_{i,i+1}^{i+1}\sigma_\eta^2$ in increasing in $\frac{1}{Var(\theta|S)-Var(\theta|\hat{s}_i,S)}$.

□

Steps from the Academic TRACE File to the Cleaned Academic TRACE Sample

Eliminate trade between a dealer and customer

As we focus on inter-dealer trade, we drop the trade between a dealer and customer. There are 11,900,080 unique trade reports on 32,795 different CUSIPs left in the dataset.

Eliminate bonds based on characteristics

I replicate the cleaning procedure in Asquith, Covert and Pathak (2019). Before eliminating and correcting trade reports, we match the TRACE dataset to the universe of corporate bonds in the Mergent FISD database. The Mergent FISD database is our source for bond characteristics such as issue size, ratings, maturity, etc. which we add to the Academic TRACE dataset. The Mergent FISD database we use includes all the bonds with an offering date between January of 1950 and January of 2010.

I drop all TRACE bonds that do not match to FISD by CUSIP. There are 11,800,641 unique trade reports on 31,682 different CUSIPs left in the dataset.

I also drop all bonds with equity-like characteristics (convertibles, exchangeables, etc.) since their equity component may be included in the bond price. There are 11,005,869 unique trade reports on 29,872 different CUSIPs left in the dataset.

I next drop all Rule 144a bonds because TRACE did not disseminate trading information on these bonds during 2002-2005.[16] There are 10,704,976 unique trade reports on 26,965 different CUSIPS left.

FISD does not report a correct issue size in some cases. For example, there are some bonds in FISD with a reported issue size of 0. I drop all bonds with a reported FISD issue size of less than 1. (offering-amt $\leq$1) There are 10,687,352 unique trade reports on 26,729 different CUSIPs left.

Eliminate trade reports because of self-reported errors

Next, we eliminate trade reports which do not take place as reported since they are later modified, cancelled, or reversed. I replicate cleaning procedure steps 1 and 2 in Dick-Nielsen (2014).

I first clean same-day corrections and cancellations. Same-day refers to corrections and

[16] One way to circumvent TRACE, which applies to publicly issued bonds, is for a firm to issue privately placed bonds (sometimes referred to as Rule 144a securities, for the section of the Securities Act of 1933 that provides exemption from registration requirements).

cancelations reported within the same reporting date (not transaction date). These can be uniquely identified by the link between the Record Count Number and Original Message Sequence Number. The Record Count Number is unique on an intra-reporting day level. There are 10,423,151 unique trade reports on 26,647 different CUSIPs left.

Remove reversals and the matching original transaction report. Reversals are cancelations reported on a later date than the date on which the orignal transaction took place. Trade reversals are identified by the As Of Indicator field by "R". Since the original trade and its reversal are reported to TRACE on different days, the Original Message Sequence Number of reversal trades do not necessarily match the Record Count Number of the original trade. Therefore, to link a reversal to its original trade, following Asquith, Covert and Pathak (2019), we match the reports using eight identifying characteristics: CUSIP, Trade Execution Date, Trade Execution Time, Reported Price, Entered Volume Quantity, Reporter ID, Contraparty ID, Buy/Sell Indicator, Buyer Capacity, and Seller Capacity. (Nick-Nielsen (2014) does not use Reporter ID, Buyer Capacity, Seller Capacity.) I called matches with these criteria a "ten-way" match.

In the dataset of reversals, these ten characteristics identify a unique observation for most of the observations. For those that are not uniquely identified, we keep the last one by the reported date and time. I keep those with a trade reported date after the trade execution date. Then we merge the dataset of original trades using the dataset of reversals. If there is only one exact match, both the reversal and its matched trade are eliminated. If there is more than one exact match, we eliminate the reversal trade and one of the matching trades. Because these multiple matching trades occur at the same time, date, price and volume, the cleaned dataset is unaffected by the choice of which matching trade reports we eliminate. There are 10,121,712 unique trade reports on 26,581 different CUSIPs left.

Not all reversals have an exact ten-way match. I then drop the same execution time requirement. Since execution time is self-reported, we assume these nine-way matches were the original trades that were meant to be reversed, and we eliminate the reversal and the matched trade following the steps above. Still, before we merge the datasets, in the dataset of reversals left, for those that are not uniquely identified by those nine characteristics, we keep the last one by the reported date and time. There are 10,113,094 unique trade reports on 26,578 different CUSIPs left.

Next, we relax the requirement that the price must be exact in a nine-way reversal and look for matches when prices are rounded to 0.01. There are 10,112,431 unique trade reports on 26,578 different CUSIPs left.

There are 16859 reversals (each of them is uniquely identified by CUSIP, Trade Execution Date, Reported Price (rounded to 0.01), Entered Volume Quantity, Reporter ID, Contraparty ID, Buy/Sell Indicator, Buyer Capacity, and Seller Capacity) that we are unable to match to an original trade. I dropped these reversal reports from the dataset.

Eliminate one of the sides for inter-dealer trade

There are multiple conventions or paths by which the transacting parties can be reported to TRACE. In addition to self-reporting, TRACE also allows another party (such as a clearing firm) to fulfill the reporting obligation of the transacting party. There are three fields that are used when trade reports do not report the transacting dealers in the trade. They are Reporter Give-up ID, Contraparty Give-up ID, and Locked-in Trade Identifier. In a Give-up trade, a clearing firm can submit a report on behalf of either of the transacting dealers. When this is the case, either the Reporter Give-up ID or Contraparty Give-up ID is populated by the ID of the transacting dealer. To correctly identify the transacting parties in these trades, we replace the Reporter ID (Contraparty ID) with the Reporter Give-up ID (Contraparty Give-up ID).

The other case where TRACE allows a variance on its reporting requirements is a locked-in trade report. In a locked-in trade report, the reporting party submits the trade report on its own behalf as well as on behalf of the contraparty. That is, there is only one trade report, rather than a separate report from the buying dealer and selling dealer. When the Locked-in Trade Identifier is checked, the reporter ID and the contraparty ID are the same, but one give-up field is populated. For each trade report where the Locked-in Flag is marked, we follow the convention in the paragraph above for the give-up fields and modify the Reporter or Contraparty ID fields in the existing trade report.

For regular trade (Regular Trade Identifier is “x”), we adopt the convention of preserving the sell-side report. For locked-in trade, the trade report is kept. There are 5,155,102 unique trade reports on 25,239 different CUSIPs left.

Address trade splitting

I basically replicate the procedure in Asquith, Covert and Pathak (2019). Dealers might report one trade in multiple pieces. To deal with trade splitting, we aggregate all of the reports with the same CUSIP, Execution Date, Price, Reported ID, Contraparty ID, Buy/Sell Indicator, Buyer Capacity, and Seller Capacity. There are 4,889,149 unique trade reports and 25,239 different CUSIPs left.

Eliminate trade reports with price or volume issues

I basically replicate the procedure in Asquith, Covert and Pathak (2019). Some trade prices are vastly out of line with other prices for the bond during the same period. The reference prices are the median prices of the same bond traded in the same month. A trade price is vastly out of line if the bond price differs by more than $20 per bond. I eliminate the reports with price vastly out of line. There are 4,885,676 unique trade reports and 25,239 different CUSIPs left.

The procedure above does not eliminate the report if there is not another trade report in that month. Therefore we drop trade reports less than the 0.01 percentile and greater than 99.99 percentile of all trade prices in the sample. Next, we eliminate trades when volume is less than the 0.01 percentile and greater than 99.99 percentile of all trade volumes in the sample. There are 4,883,688 unique trade reports and 25,181 different CUSIPs left.

Eliminate if the trade is under special circumstances, or the traded asset is an equity linked note, or the trade is not a cash sale

I replicate Dick-Nielsen (2014).There are 4,865,045 unique trade reports and 25,101 different CUSIPs left.